

Decoding long-duration GWs from BNS with machine learning: parameter estimation and equations of state



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Link to DCC

Abstract & Take-away:

- We demonstrate the feasibility of analyzing **3-hour-long GW data from BNS** with machine learning, achieving **full parameter estimation (PE)** and **constraining the equations of state (EOS)** of neutron stars **within $O(1)$ seconds**.
- We employ multi-banding and heterodyning for data preprocessing, and normalizing flows for PE and EOS inference. The results are validated against analytical predictions since full PE for such long signals is prohibitively slow.

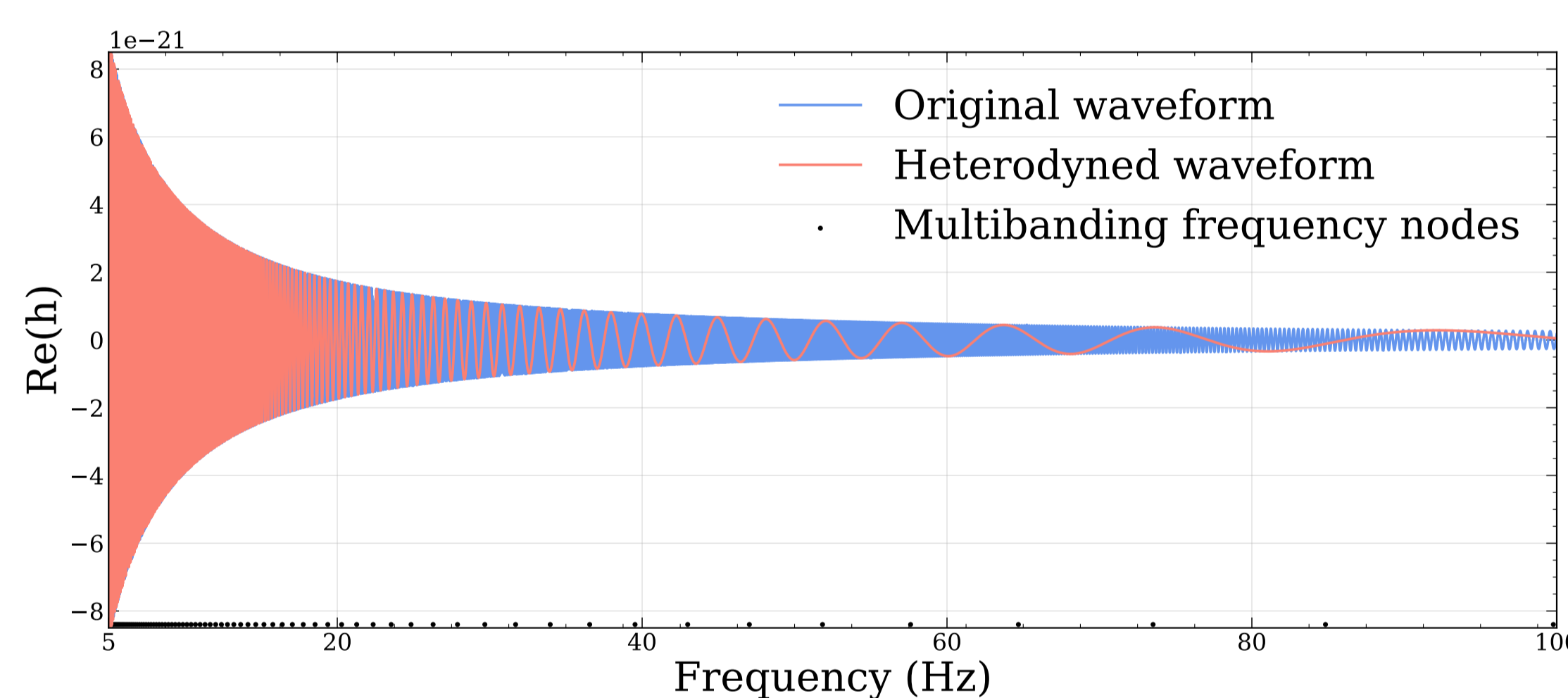
1. Introduction

- **Future:** Third-generation (3G) GW detectors networks, including Einstein Telescope (ET) and Cosmic Explorer (CE), are expected to detect **$>10^5$ BNS events per year** [1] with extended **duration up to hours** starting from ~ 5 Hz and with increased signal-to-noise ratio (SNR).
- **Problem:** PE for long-duration, high-SNR events using stochastic sampling is prohibitively slow. [2-3] investigated into subsets of the task (ignoring the Earth rotation or ignoring precession), requiring **>1000 CPU hours per event**. In this way, PE alone would cost **millions of dollars** in electricity charges per year, and it is **not environmentally friendly!**
- **Solution:** **Machine learning** is proven to be a promising way of fast PE [4]. In this work, we train neural density estimators based on normalizing flows to infer BNS source parameters and EOS rapidly with minimal hardware and time costs, enabling catalog-level analysis for long BNS signals.

2. Data Preprocessing

- **Why:** Duration=12000s and frequency band 5-1024Hz \rightarrow **12M data points**. Effective data compression is crucial to analysis of such long data.
- **Multibanding:** adaptively selects frequency nodes and resolutions based on the frequency evolution of CBC sources, ensuring that each band's resolution is precisely tuned to the needs of BNS signals. **12M \rightarrow 6000 data points, compression ratio ~ 2000** .
- **Heterodyning:** BNS waveform is highly oscillatory, making data compression inefficient. Following [5], we heterodyne the signal to **reduce the oscillations**.

Figure 1: Illustration of BNS frequency-domain waveforms before (blue) and after (orange) heterodyning. Waveforms are truncated at 100Hz for better illustration. Black dots denote the frequency nodes of the multibanding scheme: the bands are defined between nodes.



3. Neural Density Estimation

- **For parameter estimation,** we first extract the principal components of the preprocessed data by SVD and use an embedding network to further compress the data. We tried two types of embedding networks: MLP Residual Network (MLPResNet) and Transformer (ViT). The data is then used by a conditional normalizing flow to generate posterior samples.
- **For equations of state,** following [6], we train an independent normalizing flow conditioned on the BNS mass and tidal parameters to infer a compact form of EOS, which can be converted to the original EOS by an autoencoder.

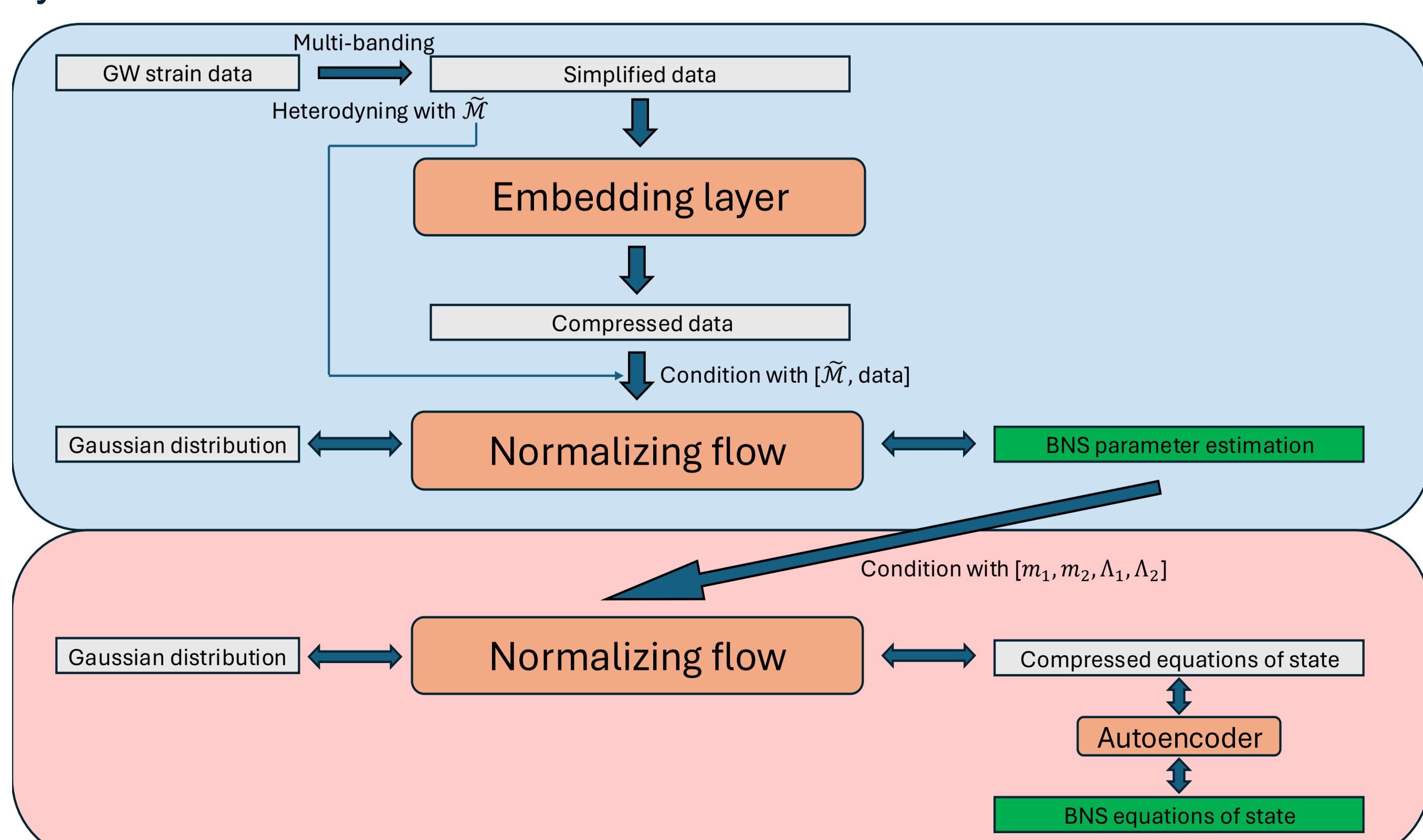


Figure 2: The structure of our neural networks.

4. BNS Parameter Estimation

- **Prior:** **1ET + 2CE** network + stationary Gaussian noise at their design sensitivities. We ignore signal overlaps. SNR range is set to 20-50. Detector frame chirp mass 2- $2.1M_{\odot}$, which corresponds to $\sim 1.4M_{\odot}$ in the source frame. A higher SNR model is still being trained.
- **Training:** >60 million intrinsic parameters, random extrinsic parameters.
- **Speed:** Models with different embedding layers give consistent results (Fig.3 left). It takes $\sim 0.2s$ to generate 5000 samples on RTX 3080.
- **Precision:** since full PE is prohibitively slow, we **compare statistical errors** with Fisher matrix forecast and **compare skymaps** with a fast localization algorithm SealGW [7]. The width of their statistical errors mostly agree (Fig.3 top right). A potential issue: Fisher matrix may not be accurate for the 17D problem.
- **Accuracy:** We present the **p-p plot** and **p-values** in Fig.3 lower right.
- **Future work:** Higher SNR, overlapping signals, noise variations...

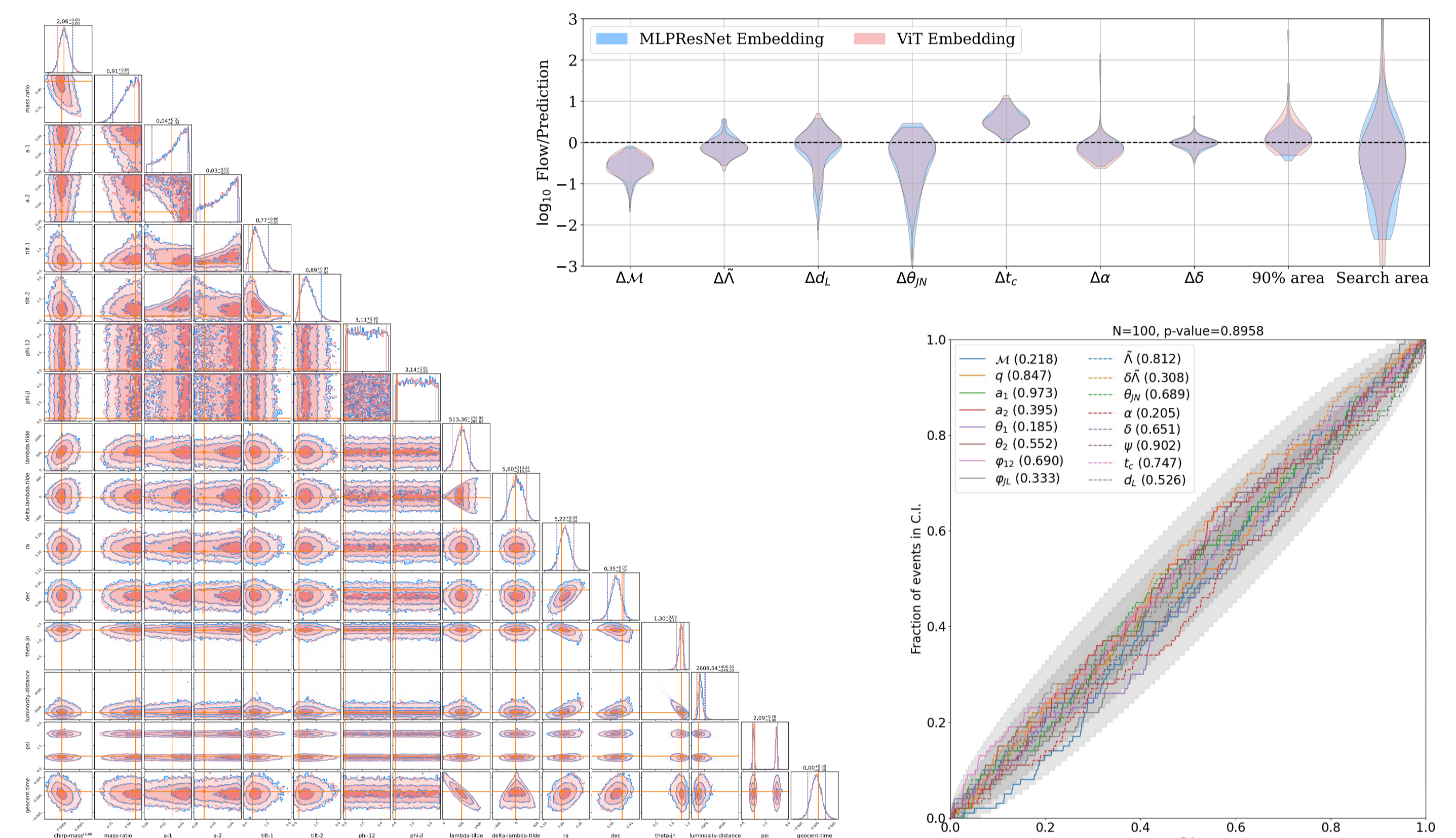
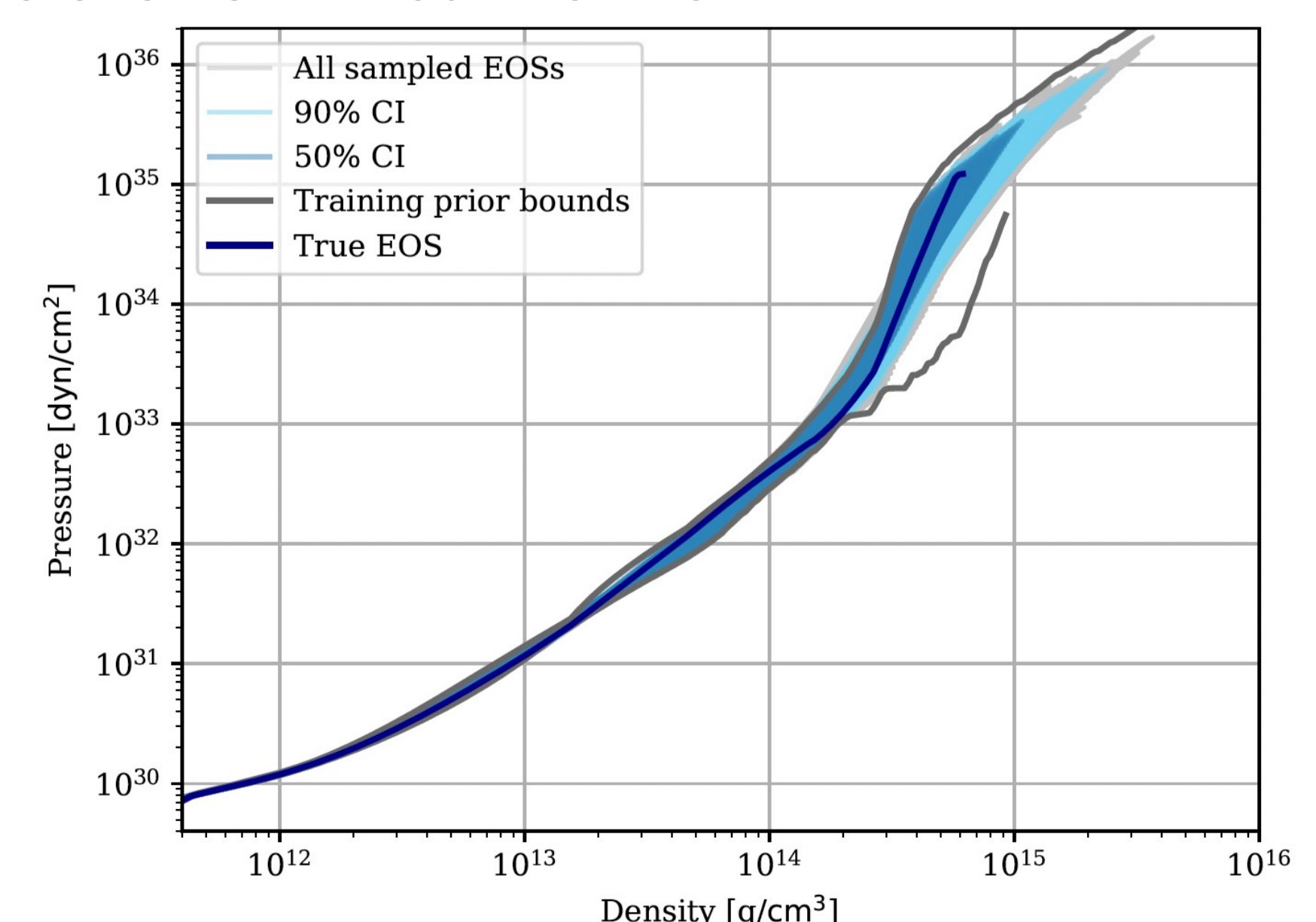


Figure 3: Parameter estimation results. **Left:** An example corner plot. Blue contours represent the model with MLPResNet embedding while the red represents ViT embedding. **Top right:** Ratios between the statistical errors given by flow models and given by Fisher matrix/SealGW. The dashed line indicates the location where flow models perfectly match analytical results. **Lower right:** The p-p plot of the ViT-embedding model with p-values shown. The MLPResNet model gives similar results.

5. Inferring Equations of State

- **Training:** Following [6], a normalizing flow was trained on a compressed set of EOS from CUTER [8] consisting of a meta-model + piecewise polytrope structure. These EOS were compressed to a 12D representation by a convolutional autoencoder.
- **Injection test:** **Given a true EOS, a BNS event was simulated** and PE was performed. The posterior samples were then passed to the normalizing flow to generate an EOS posterior. This was decoded into a pressure-density relationship using the autoencoder (Figure 4).
- **Future work:** combine multiple events with our workflow...

Figure 4: We present 50% and 90% confidence intervals (CIs) in dark blue and light blue respectively, as determined by the autoencoded latent space representation of the EOS and presented here as pressure as a function of density. The injected EOS is given in dark blue with the training prior bounds given in grey, illustrating the most stiff and most soft EOSs in our training data set.



6. References

- [1]. Branchesi+ 2303.15923, [2]. Smith+ 2103.12274, [3]. Wong+ 2302.05333, [4]. Dax+ 2106.12594, [5]. Dax+ 2407.09602, [6]. McGinn+ 2403.17462, [7]. Hu+ 2110.01874, [8]. Davis+ 2406.14906