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Systematic error accumulation in testing GR: Overlapping signals and waveform systematics

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Overview

- Predicting systematic errors in PE: overlapping signals and inaccurate waveforms
- Mock catalog for ET&CE
- Systematic error accumulation – using parameterized post-Newtonian (PPN) coefficients tests of GR as an example

Excess strains

Where systematic errors originate (arXiv:0707.2982, 2104.01897)

- Likelihood in PE: assuming noise is stationary and Gaussian, and d-h=n

$$\ln L(\vec{\theta}) = -\frac{1}{2}(d-h|d-h) = -\frac{1}{2}(n|n),$$

- Maximum likelihood (ML) estimator:

$$\partial_i \ln L \,|_{\vec{\theta}=\vec{\theta}_{\rm ML}} = (\partial_i h|d-h)\,|_{\vec{\theta}=\vec{\theta}_{\rm ML}} = 0,$$

- The statistical error caused by noise:

$$\Delta\theta^i_{\rm stat} \approx (\Gamma^{-1})^{ij}(\partial_j h_{\rm m}|n)$$

$$< \Delta\theta^i_{\rm stat} >= 0$$ Unbiased

$$< \Delta\theta^i_{\rm stat}\Delta\theta^j_{\rm stat} >= (\Gamma^{-1})^{ij}$$ Fisher matrix forecast

- If there are strains other than noise (d-h=n+δH), the ML will be biased:

$$\Delta\theta^i_{\rm sys} = (\Gamma^{-1})^{ij}(\partial_j h_{\rm m}|\delta H)$$

Our concerns

$$\Delta\theta^i_{\rm stat} \approx (\Gamma^{-1})^{ij}(\partial_j h_{\rm m}|n) \qquad \Delta\theta^i_{\rm sys} = (\Gamma^{-1})^{ij}(\partial_j h_{\rm m}|\delta H)$$

- $\Delta\theta^i_{stat}$ ∝1/SNR, $\Delta\theta^i_{sys}$ may not . When SNR increases, systematic errors may dominate

- Waveform systematics
  - Does not scale down with SNR. May already cause some tension in LVK results (2205.08448, *PRD* 106 4, 044042)
  - $\delta H_{waveform} = h_{real} - h_{model}$
- Overlapping signals:
  - Detected overlaps: $\delta H_{do} = h'_{real}(\theta_{true}) - h'_{model}(\theta_{ML}) \approx \Delta\theta'^{i}_{stat}\partial_i h'_m + \delta H'_{waveform}$ (to the lowest order)
  - Undetected overlaps (SNR<8): $\delta H_{\rm uo} = h''_{real}(\theta_{true})$

PPN, waveforms, and parameters

- Parametrized post-Newtionial (PPN) coefficient tests of GR:
  - IMRPhenomPv2 phase is characterized by a set of parameters $\{p_i\}$
    - Inspiral regime parameters: $\{\varphi_0, \dots, \varphi_7\}, \{\varphi_{5l}, \varphi_{6l}\}$
    - Phenomenological coefficients: $\{\beta_0, \dots, \beta_3\}$
    - Merger-ringdown parameters: $\{\alpha_0, \dots, \alpha_5\}$

$$\varphi_{\rm PN}(f) = 2\pi f\, t_{\rm c} - \varphi_{\rm c} - \frac{\pi}{4} + \frac{3}{128\eta}\left(\pi \tilde{f}\right)^{-5/3} \sum_{i=0}^{7} \left[\varphi_i + \varphi_{il} \log(\pi \tilde{f})\right] \left(\pi \tilde{f}\right)^{i/3}.$$

  - $p_i \to (1 + \delta\hat{p}_i)p_i$, the $\delta\hat{p}_i$ is the testing parameter. $\delta\hat{p}_i = 0$ returns to GR
- We choose $\delta\widehat{\varphi_0}$ as an example testing parameter in this work
- We perturbate $\delta\widehat{\beta_2}$ as the inaccurate waveform parameter:
  - $\delta\widehat{\beta_2} = 0$: model waveform
  - $\delta\widehat{\beta_2} = 5\times10^{-2}$: “real” waveform for current waveform modelling, mismatch ~ $10^{-4} - 10^{-3}$
  - $\delta\widehat{\beta_2} = 5\times10^{-4}$: “real” waveform for future waveform modelling, mismatch ~ $10^{-7} - 10^{-6}$
- Parameters in GR: chirp mass, mass ratio, merger time, effective spin. Other parameters are set to be perfectly known

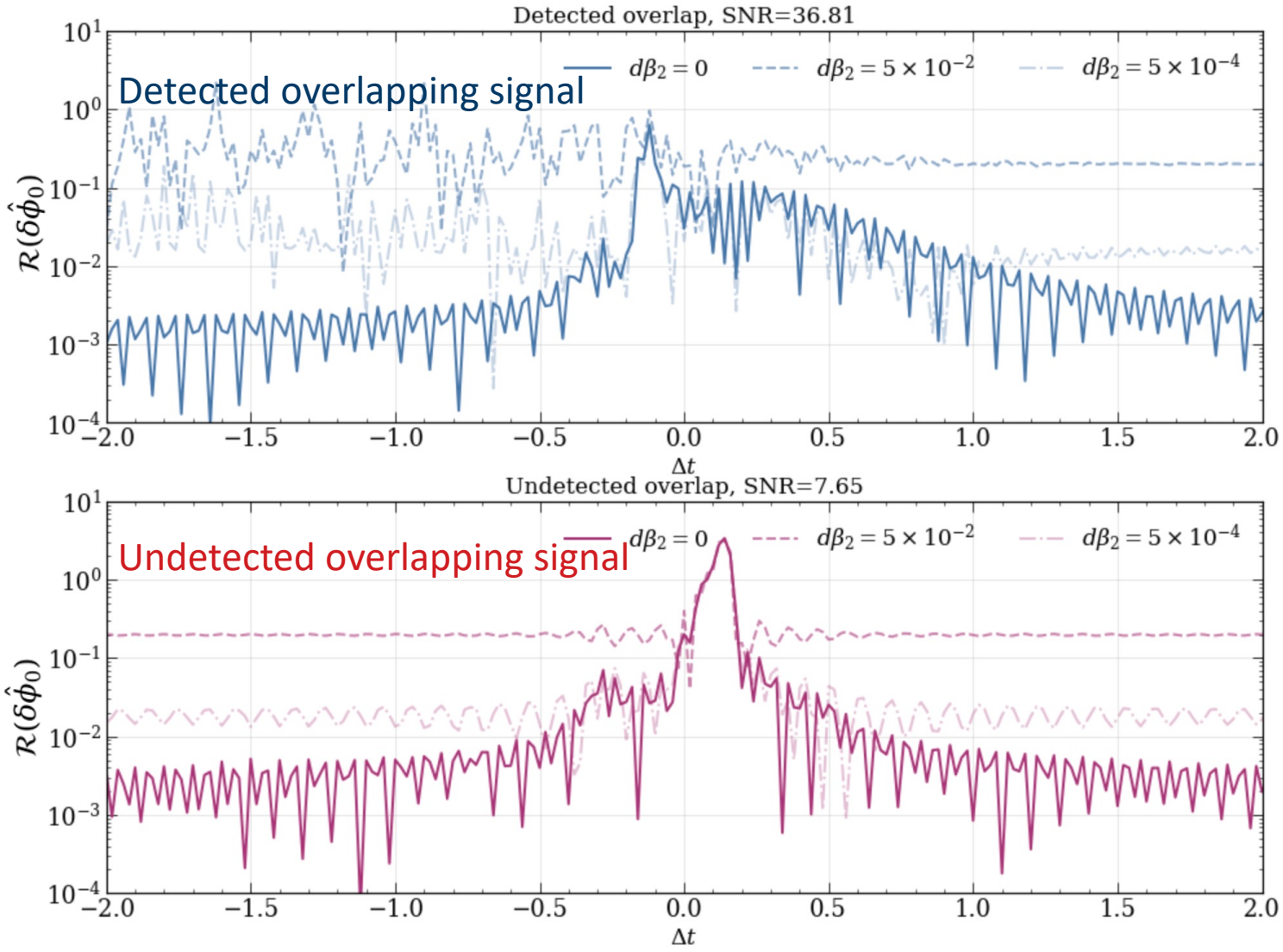
Case study

An example event

- We assume GR is the correct theory

- Define R(θ) = |sys error / stat error|
- R(θ)>1: false deviation from GR

- Oscillating, asymmetric
- $|\Delta t| < 0.5s$: overlap signal dominates
- Undetected overlaps *may* have larger impacts than the detected



Catalog simulation

- We use an analytical approximation for merger rate density (Oguri 2018)

$$\frac{R_{\rm GW}(z)}{\rm Gpc^{-3}yr^{-1}} = \frac{a_1 e^{a_2 z}}{e^{a_3 z} + a_4}, \qquad R^{\rm obs}_{\rm GW}(z) = R_{\rm GW}(z)\,\frac{dV_c}{dz}(z).$$

- Scaled according to local merger rate estimation from GWTC-2

- BBH
  - mass: power law + peak model in GWTC-2 population inference
  - aligned spin
- BNS
  - All BNS have the same intrinsic parameters
  - 1.45+1.4 solar mass, zero spin, tidal deformability parameter=425
- Isotropic inclination and sky direction
- We will test GR with all BBH events. BNS only appears as overlap background

Catalog simulation

BBH and BNS: summary

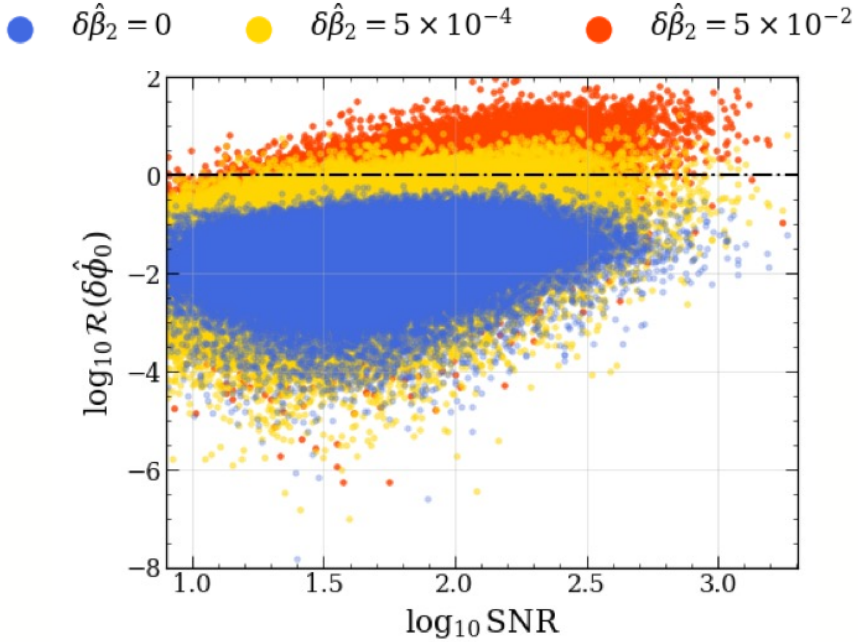
- Overlap: $|\Delta t|$<4s
- 88k observable BBH per year, among which ~80k are detectable (SNR>8) for ET+CE network

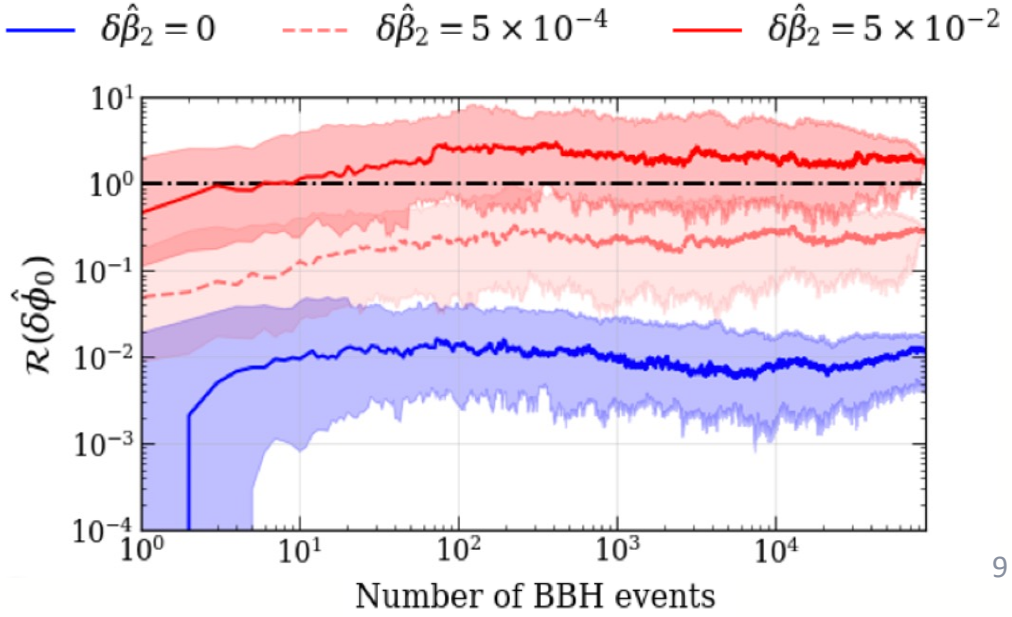
# of observable binaries		Detected overlaps on BBH events		Undetected overlaps on BBH events	
BBH	BNS	# of overlaps	# (fraction) of events	# of overlaps	# (fraction) of events
88300	1144354	0	73200 (84%)	0	76270 (87%)
		1	13125 (15%)	1	10461 (12%)
		2	1093 (1.2%)	2	721 (0.82%)
		3	67 (0.077%)	3	35 (0.040%)
		4	2 (0.0023%)		

Catalog tests I

Multiplying likelihood (=posterior)

- Multiply likelihood directly => Assuming testing parameter is the same across the catalog
- Statistical error ~ N^(-1/2); Systematic error ~N^(-1/2+ ε), The ε>0 comes from non-zero mean of Gaussian distribution when multiplying posteriors.
- Error ratio may increase to 1 as number of events increases
- Waveform systematic is more problematic than overlap signals

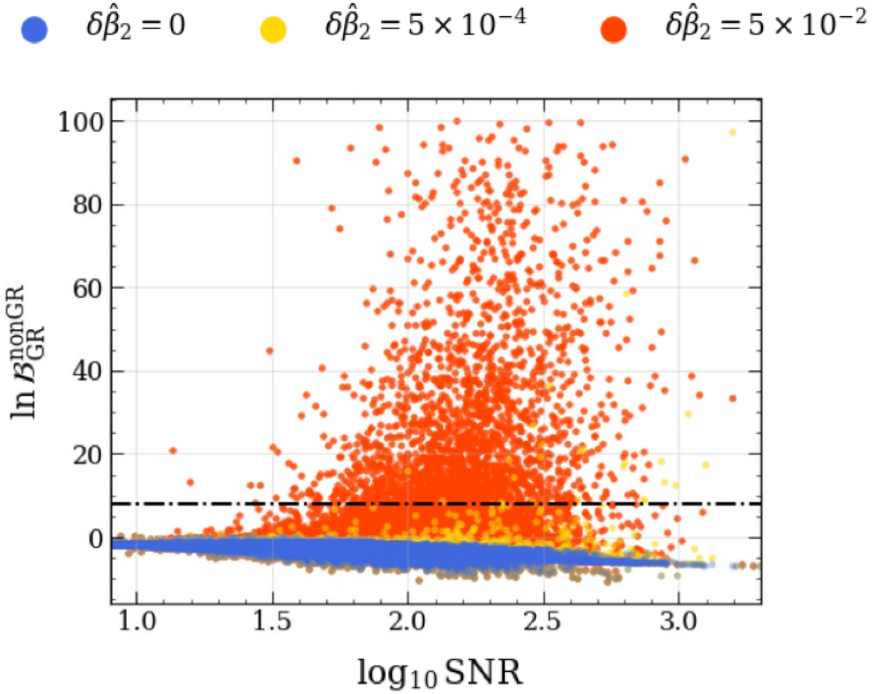


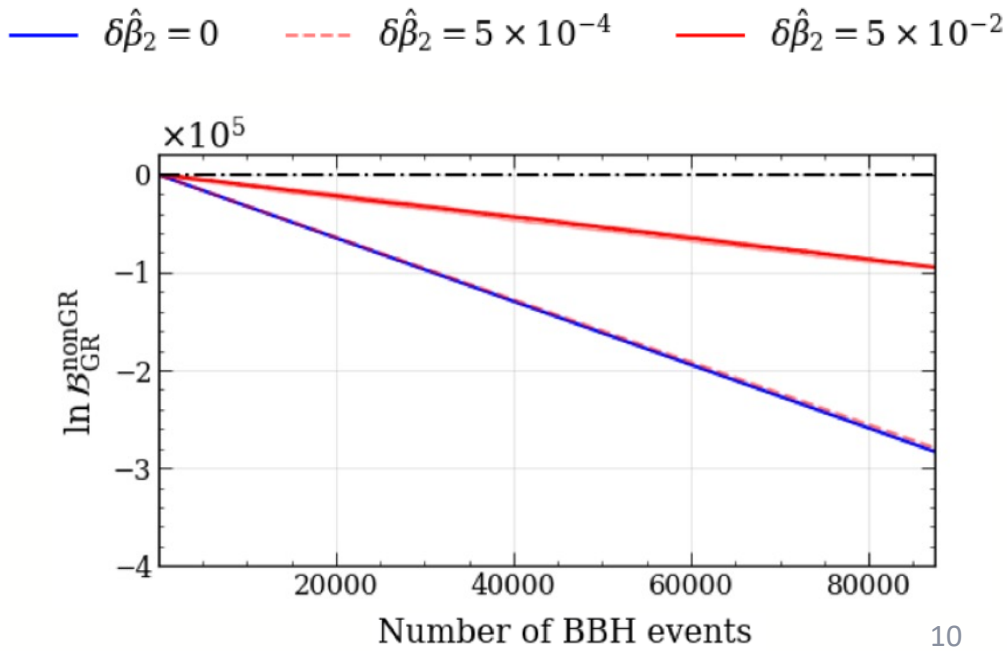


Catalog tests II

Multiplying Bayes factors

- Marginalize posterior ->Bayes factor (BF) and multiply BFs of multiple event
  - => Do not assume the same testing parameter
- As error can accumulate, the correctness (BFs that are in favor of GR) also accumulate



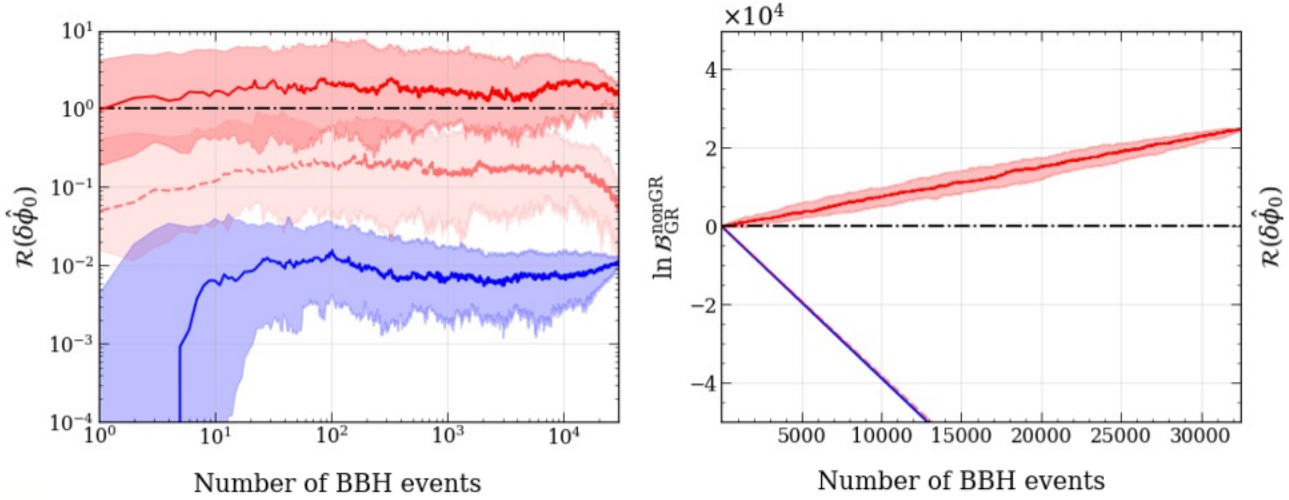


Golden events

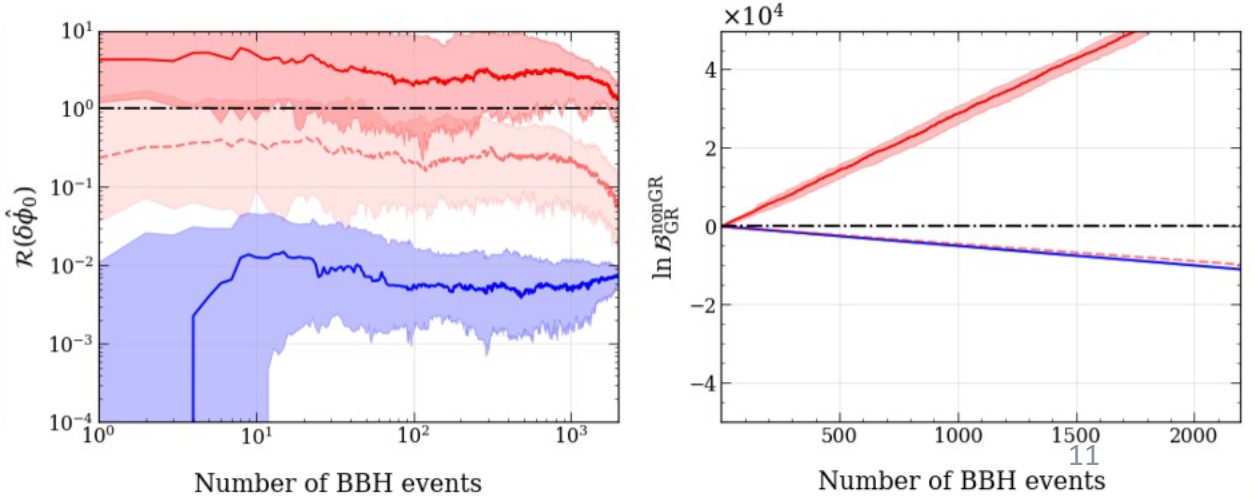
Testing GR with selected high SNR and clean events

- SNR>50 or 200 & No detected overlapping signal
- Golden or rotten?
  - High SNR events are more likely to show “evidences” of deviation of GR due to systematic errors
  - Every high SNR event need to be carefully investigated
- Improving waveform accuracy could effectively avoid this problem

SNR>50

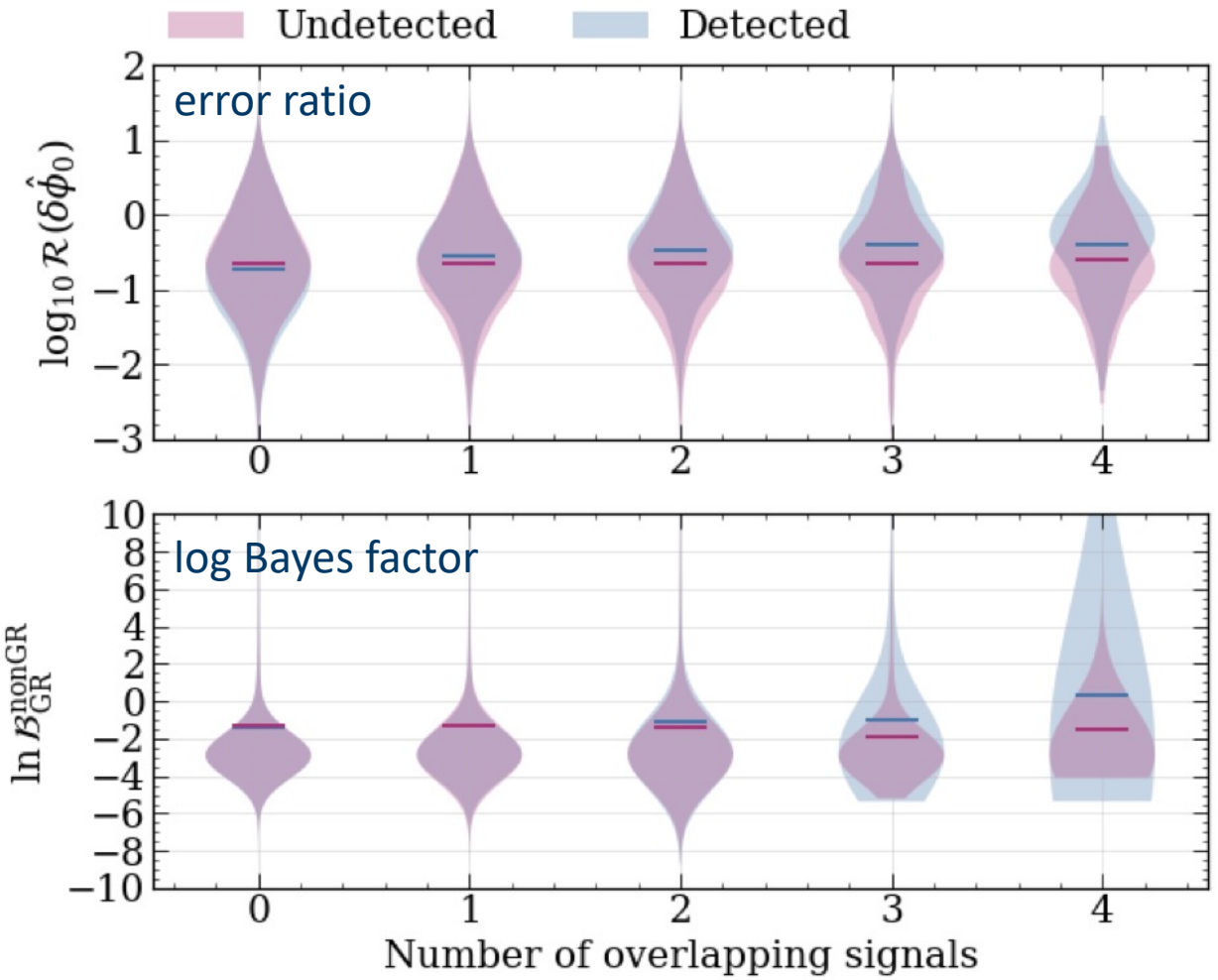


SNR>200



Accumulate with number of overlap signals

- Simulated a higher merger rate catalog
- Calculate systematic errors induced by undetected+waveform and detected+waveform ($\delta\widehat{\beta_2} = 5\times10^{-2}$)
- As number of overlapping signals increases:
  - Both systematic errors go up
  - Errors from detected overlapping signals increases faster due to more and more inaccurate signal subtractions, i.e., effects of waveform systematics are magnified



Summary

- Waveform systematic vs (detected/undetected) overlap signals:
  - Detected overlapping signals magnifies waveform systematics errors
  - Overlapping signals do not always exist, it is inaccurate waveform that keeps contributing to systematic error
- Catalog tests of GR:
  - Systematic errors do accumulate when we combine results from multiple events
  - Multiplying posteriors and BFs could both lead to false deviations
  - Golden events are more vulnerable to false deviations: systematic error dominates at high SNR scenario
  - An accurate waveform model is effective at preventing false deviations in most cases
- The idea of error accumulation is universal: testing GR, cosmology, population inference, EoS of neutron stars, …

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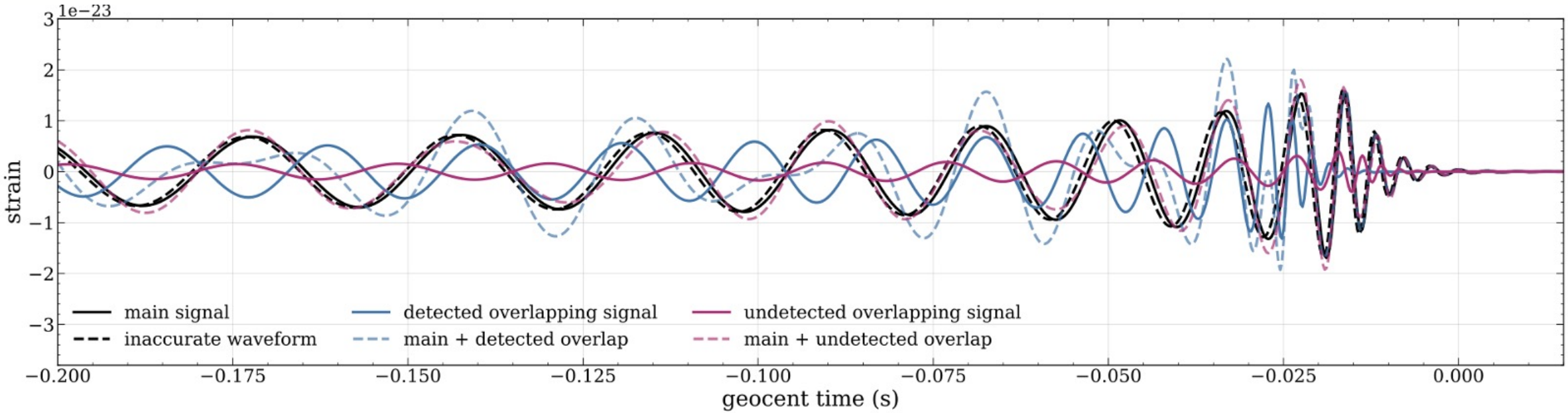
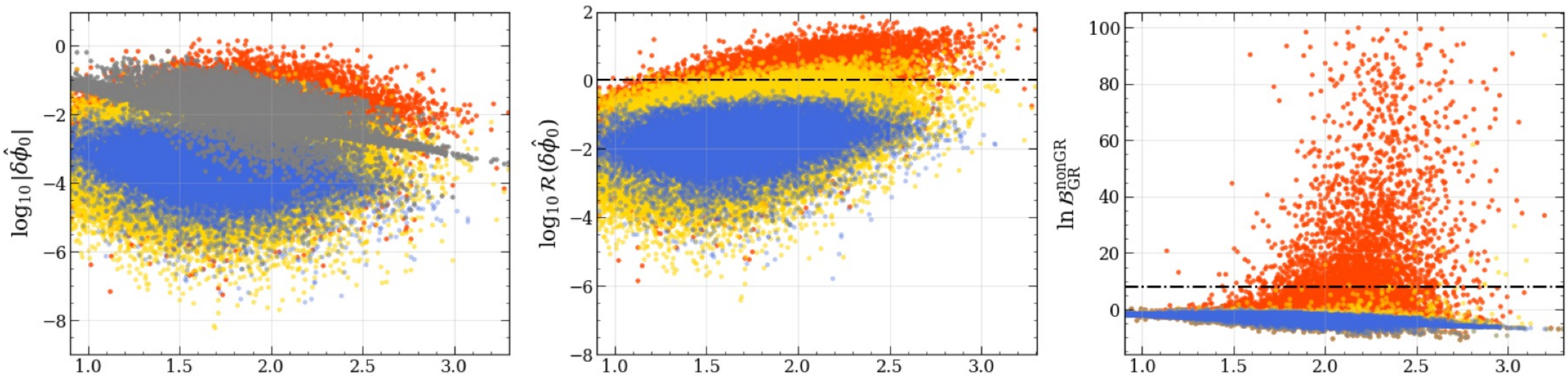
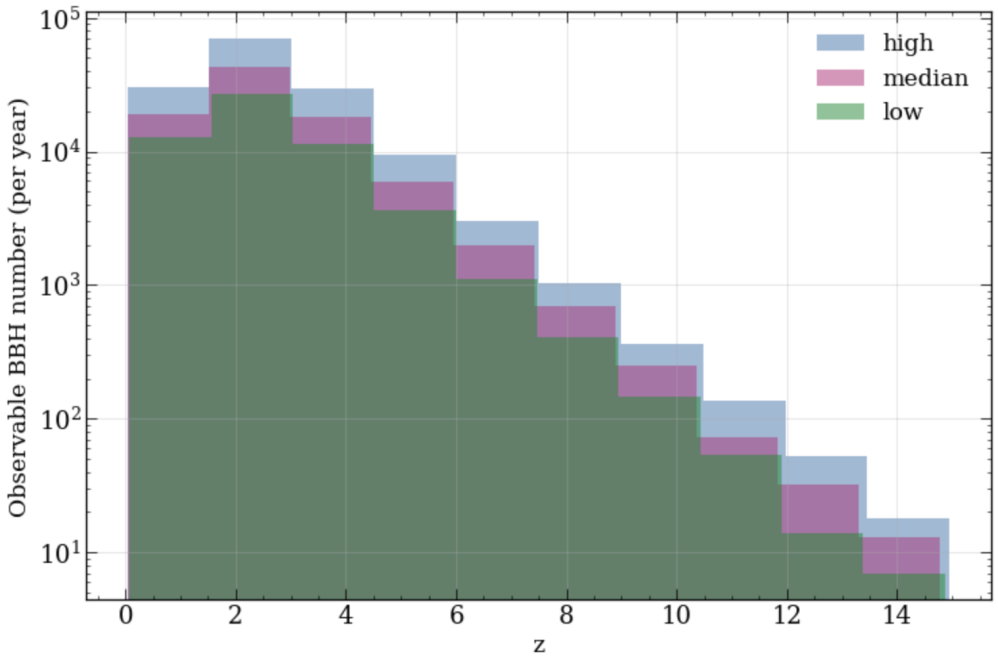
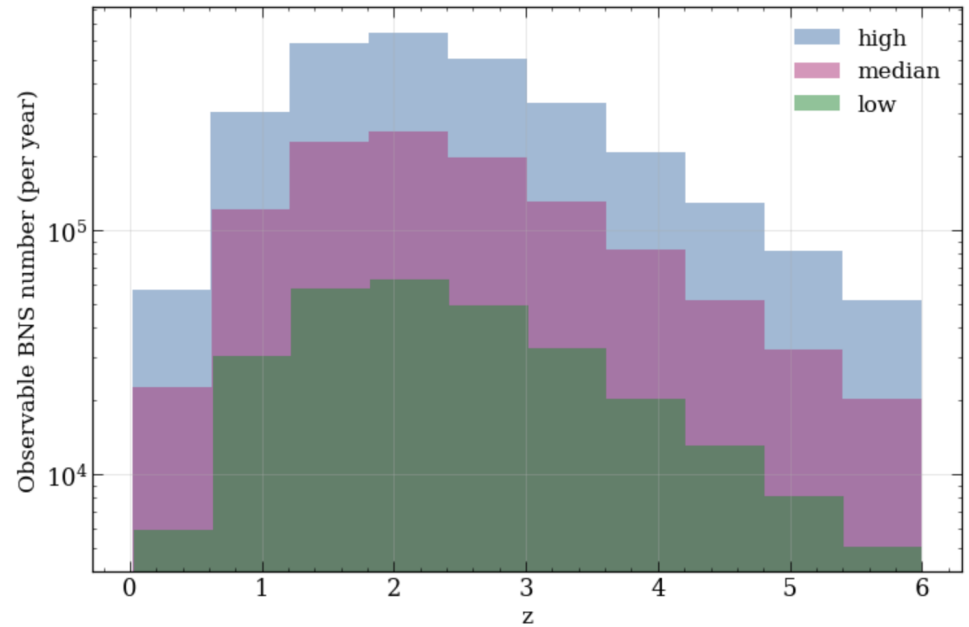

1e−23
3
2
1
0
−1
−2
−3
strain
main signal
inaccurate waveform
detected overlapping signal
main + detected overlap
undetected overlapping signal
main + undetected overlap
−0.200
−0.175
−0.150
−0.125
−0.100
−0.075
−0.050
−0.025
0.000
geocent time (s)

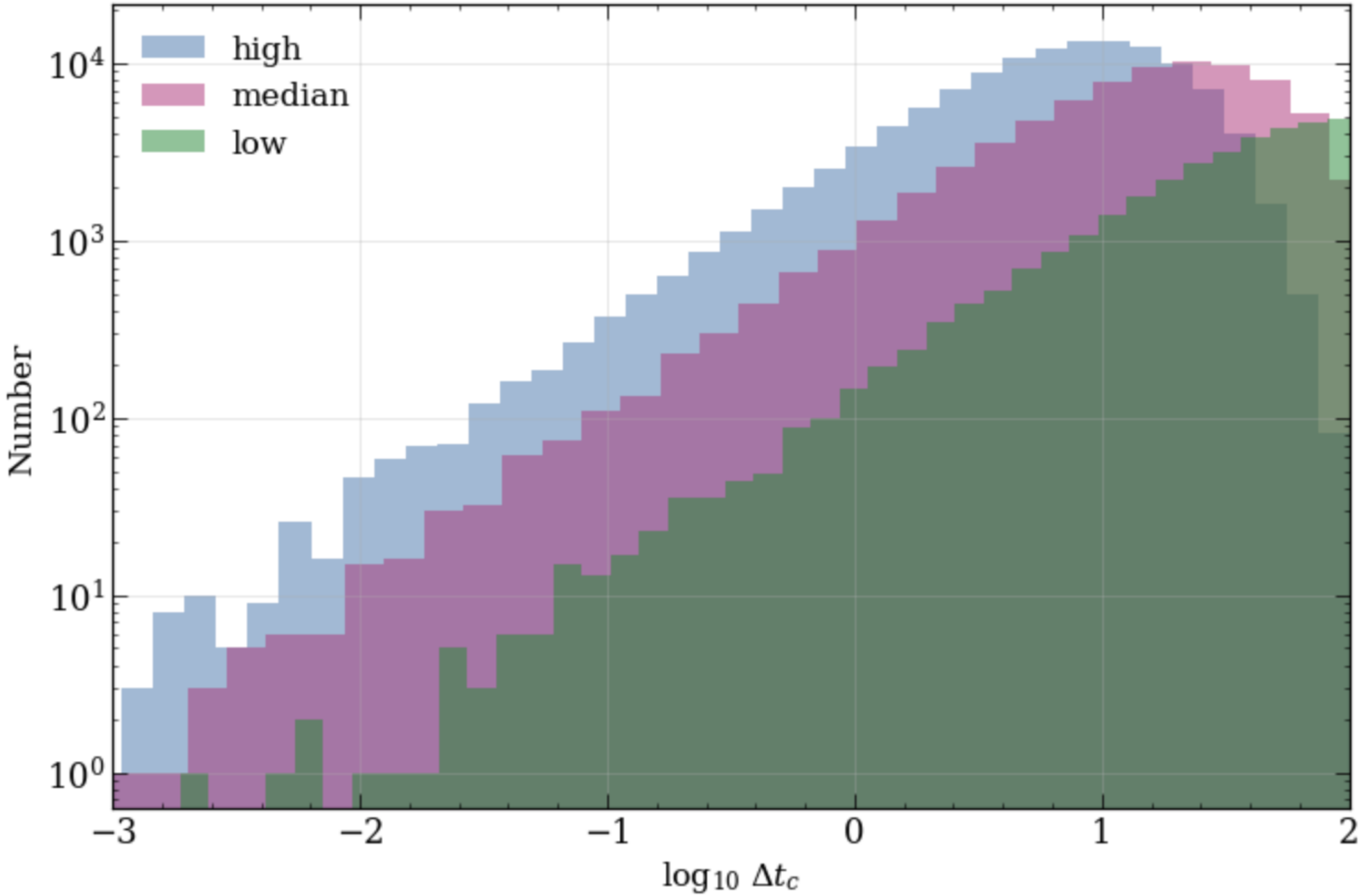

$\log_{10}|\delta\hat{\phi}_0|$
$\log_{10}\mathcal{R}(\delta\hat{\phi}_0)$
$\ln \mathcal{B}^{\rm nonGR}_{\rm GR}$
1.0
1.5
2.0
2.5
3.0

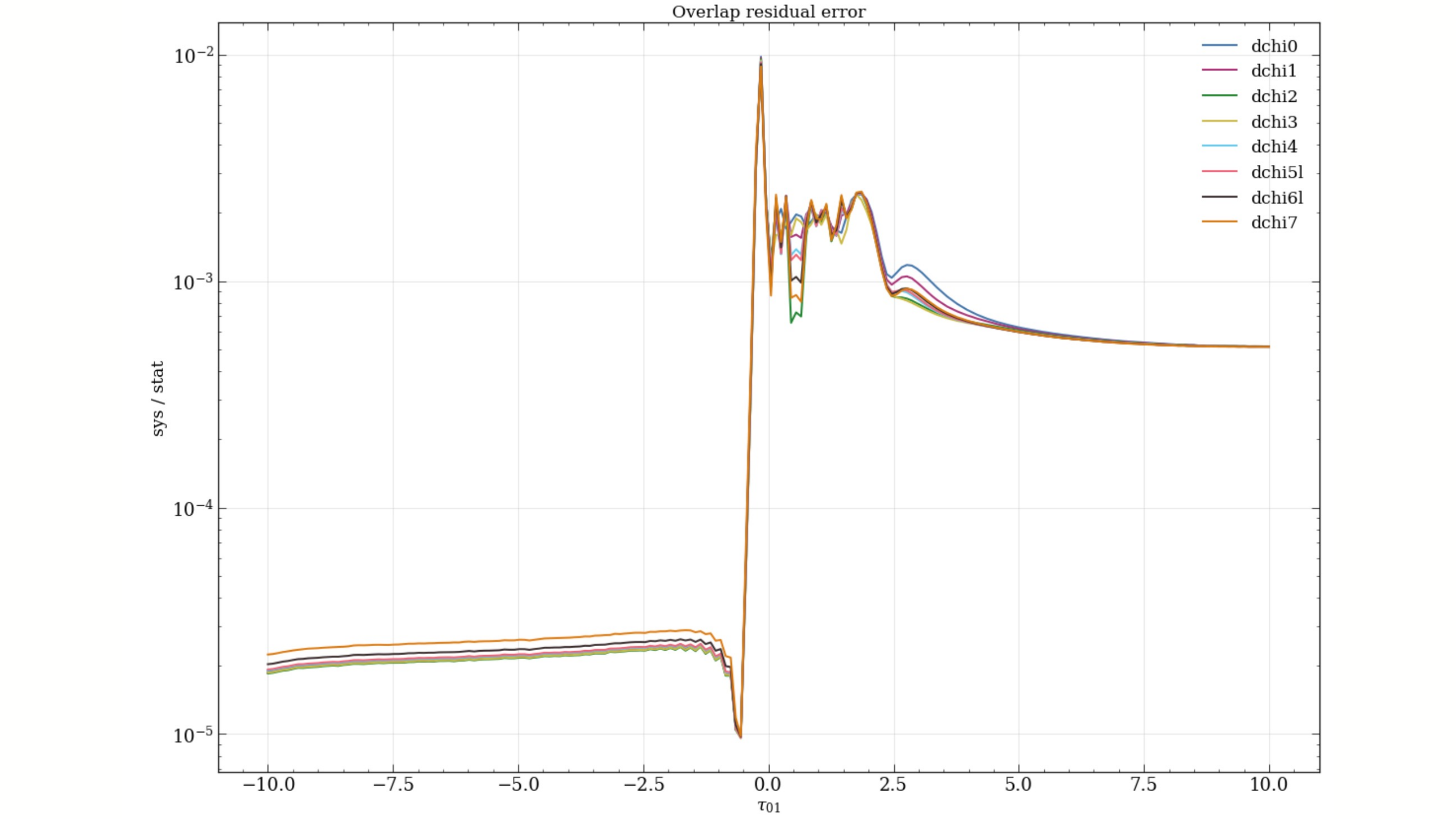
Local BBH rate: 15.3, 23.9, 38.8 Gpc^-3y r^-1
Total observable BBH number: 56526, 88300, 143349 yr^-1



Local BNS rate: 80, 320, 810 Gpc^-3y r^-1
Total observable BNS number: 286088, 1144354, 2896647 yr^-1




high
median
low
Number
10^0
10^1
10^2
10^3
10^4
−3
−2
−1
0
1
2
$\log_{10} \Delta t_c$


Overlap residual error
dchi0
dchi1
dchi2
dchi3
dchi4
dchi5l
dchi6l
dchi7
10^{-2}
10^{-3}
10^{-4}
10^{-5}
sys / stat
−10.0
−7.5
−5.0
−2.5
0.0
2.5
5.0
7.5
10.0
τ_{01}