On the model waveform accuracy of gravitational waves

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Overview

❑ A new approach to evaluate GW waveform accuracy

- By looking into difference between two waveform models
- Free from the unknown true waveform or numerical relativity (NR) simulations

❑ Applied to…

- GWTC-3 and GWTC-2.1 PE samples: How was IMRPhenomXPHM and SEOBNRv4PHM's performance?
- The relation between waveform difference and posterior difference
- Good and bad regions in the parameter space & future accuracy requirements

Assessment of one waveform model

- Can detectors distinguish it from the real one?

- “Accurate enough”: the detector can not distinguish it from the real waveform
- Construct such a waveform family for plus polarization: (Lindblom+, Phys. Rev. D **78**, 124020, 2008)

$$H_1^+(\lambda) = (1-\lambda)h_0^+ + \lambda h_1^+ = h_0^+ + \lambda\delta h_1^+, \quad 0 < \lambda < 1,$$ h0: real waveform, h1: model waveform

- Distinguishing waveforms <=> measuring λ

$$\sigma_\lambda^{-2} = \left(\frac{\partial h^+}{\partial\lambda} \,\middle|\, \frac{\partial h^+}{\partial\lambda}\right) = (\delta h_1^+ \mid \delta h_1^+).$$

$$(a \mid b) = 4\int_0^{+\infty} \frac{a^*(f)b(f)}{S_n(f)} df$$

- If the error of measuring λ is greater than its domain of definition (also the parametric distance between real and model waveforms), the detector can not distinguish

$$\|\delta h_1^+\|^2 = (\delta h_1^+ \mid \delta h_1^+) < 1.$$

- It shows: waveform error should lie within a unit ball in the inner-product space

Assessment of waveform pair

- Eliminate the unknown real waveform

$$\|\delta h_1^+\|^2 = (\delta h_1^+ \mid \delta h_1^+) < 1. \qquad \delta h_1^+ = h_1^+ - h_0^+$$

- The calculation of δh_1^+ needs the real waveform, which we don't know
- Use Numerical Relativity (NR) simulations as real waveform, but the number of NR simulations is limited
- Introduce another waveform model h_2, pair it with h_1

$$\begin{aligned}\Delta^+ &= \delta h_1^+ - \delta h_2^+ \\ &= (h_1^+ - h_0^+) - (h_2^+ - h_0^+) \\ &= h_1^+ - h_2^+.\end{aligned}$$

Real waveform is cancelled!

- Assume two waveforms are both accurate enough, we have

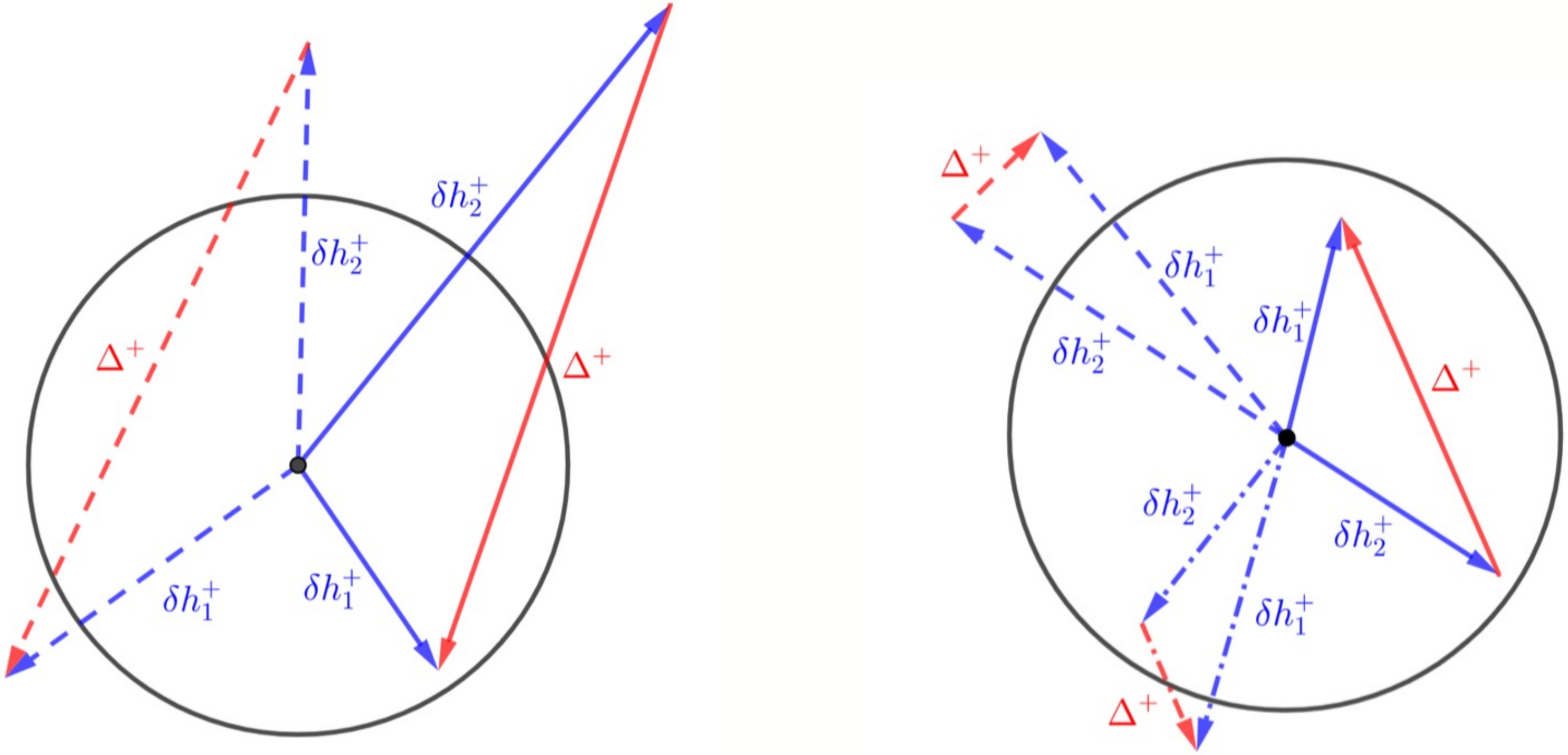
$$\|\Delta^+\| \leq \|\delta h_1^+\| + \|\delta h_2^+\| < 2.$$

- If we find $|| \Delta^+ || > 2$, at least one of the waveforms is not accurate enough. It's a necessary condition of "a pair of waveform models are both accurate".

Assessment of waveform pair

- An illustration of all possible cases

- If we find $|| \Delta^+ || > 2$, at least one of the waveforms is not accurate enough
- Even though we have got $|| \Delta^+ ||$, we don't know the real situation (possibilities are plotted in different line styles.)



Case I: $\Delta^+ > 2$ **Case II:** $\Delta^+ < 2$

Assessment of waveform pair

$$\|\Delta^{+}\| \leq \|\delta h_1^{+}\| + \|\delta h_2^{+}\| < 2.$$

- Extend to detector response:

$$\|\Delta\| \leq \|\delta h_1\| + \|\delta h_2\| < 2(|F_{+}| + |F_{\times}|).$$

- Extend to detector network:

$$\mathbf{C} = (\mathbf{D}|\mathbf{B}) \Rightarrow C_{jk} = \sum_{p=1}^{n} (D_{jp} \mid B_{pk})$$

$$\|\boldsymbol{\Delta}_{\mathrm{det}}\| = (\delta \mathbf{h}^{\mathbf{T}}|\delta \mathbf{h}) = \sum_k (\delta h^{(k)}|\delta h^{(k)})$$
$$= \sum_k \left(\Delta^{(k)}\right)^2 < 2\sum_k (|F_{+}^{(k)}| + |F_{\times}^{(k)}|)$$

- To sum up:

$$\Delta'^{(k)} = \frac{\Delta^{(k)}}{|F_{+}^{(k)}| + |F_{\times}^{(k)}|}, \quad \boldsymbol{\Delta}'_{\mathrm{det}} = \frac{\boldsymbol{\Delta}_{\mathrm{det}}}{\sum_k (|F_{+}^{(k)}| + |F_{\times}^{(k)}|)}$$

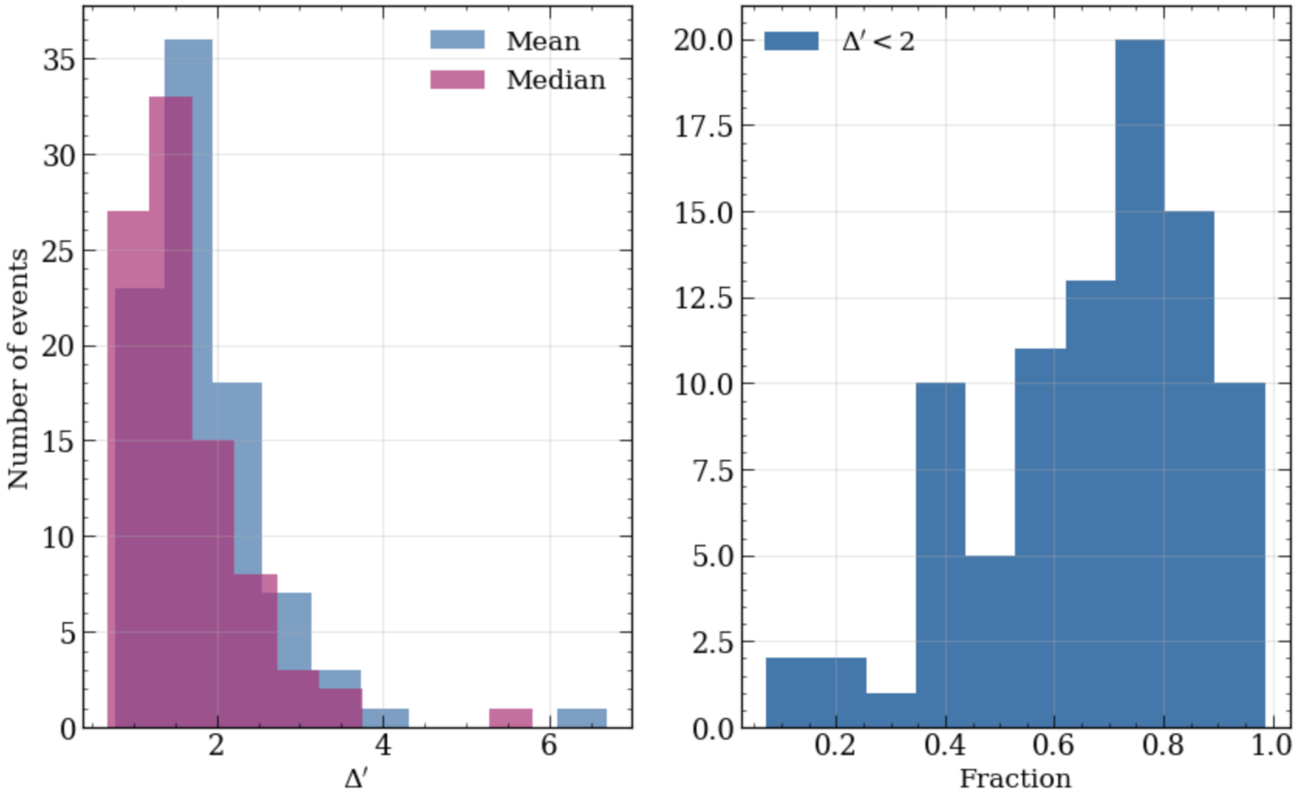
They should be less than 2 if both models are accurate!

Applying to PE samples

- Overview: histograms

- For each event, calculate $\Delta^{'}_{net}$ for the mixed posterior samples from IMRPhenomXPHM & SEOBNRv4PHM
- Calculate mean, median of $\Delta^{'}_{net}$ for each event (left panel)
- Calculate fraction of $\Delta^{'}_{net} < 2$ samples for each event (right panel)

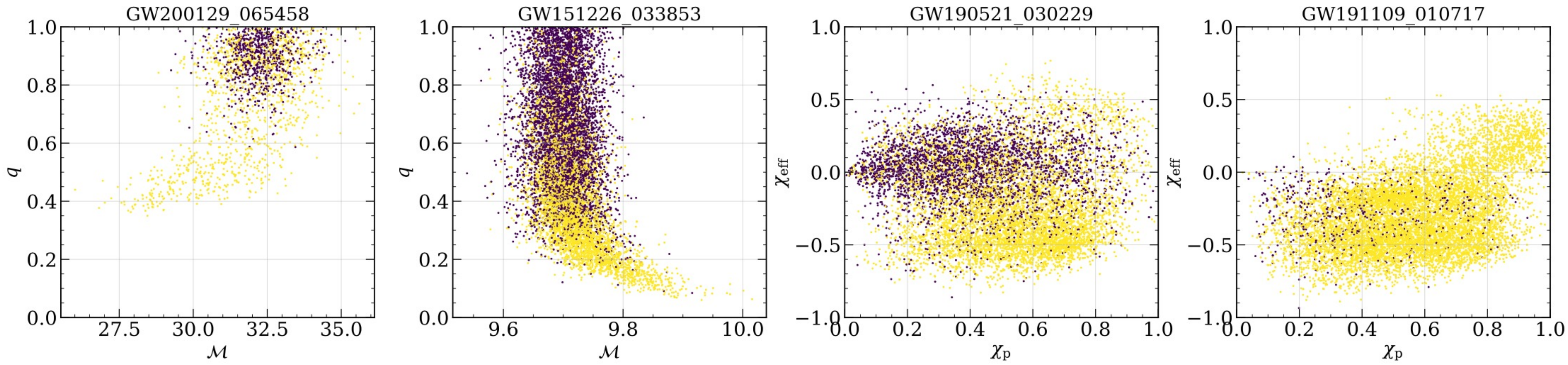
- There are several events having “worse” performance compared to the others



Applying to PE samples

- Overview: distribution in mass and spins

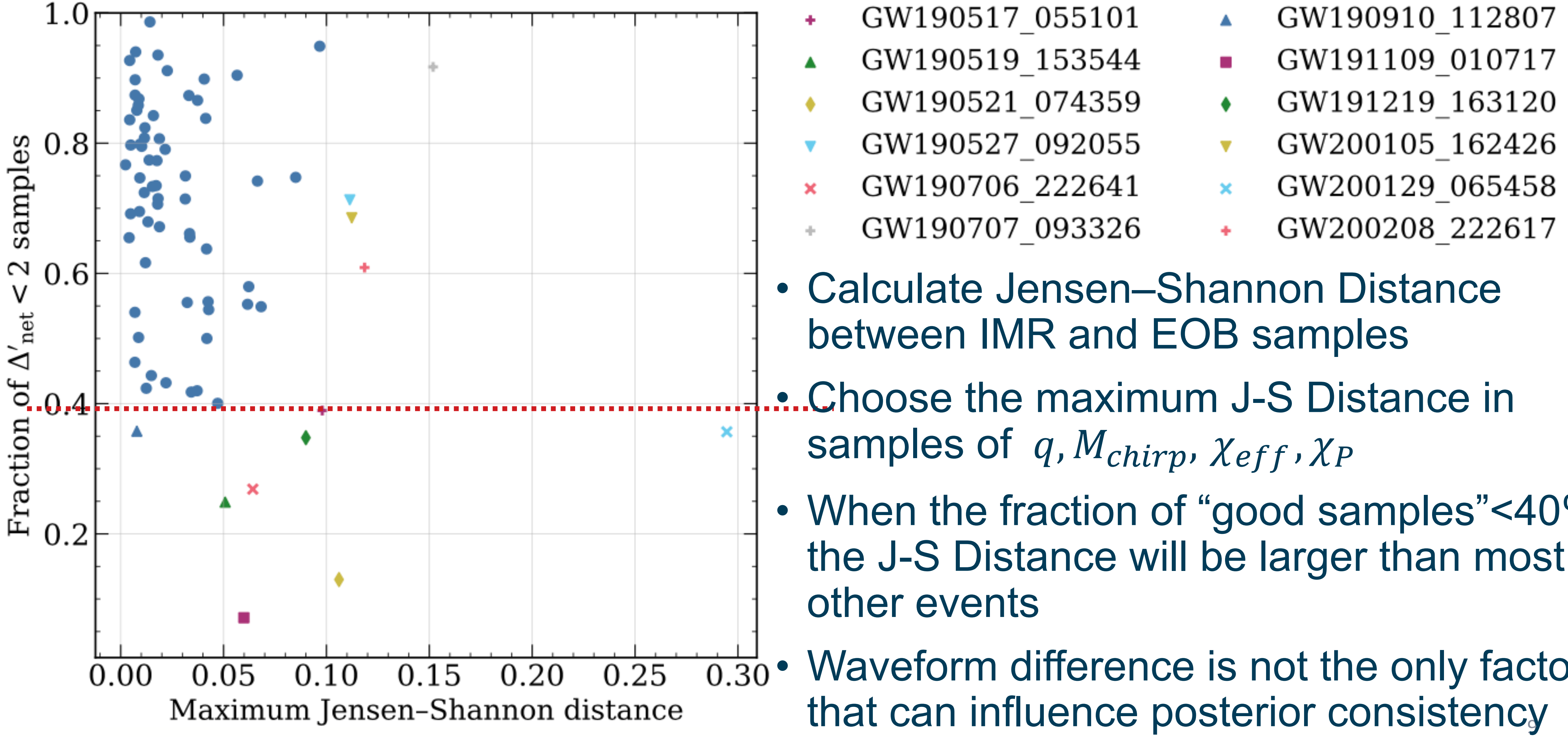
- Yellow points: $\Delta^{'}_{net} > 2$ samples ("bad" samples)
- Purple points: $\Delta^{'}_{net} < 2$ samples
- Accuracy becomes worse when mass ratio decreases or spins increase
- Using $\Delta \propto SNR$, for 3rd-gen detectors (SNR 30~1000), the model mismatch from true waveform should be improved by 3-4+ orders of magnitude (consistent with Pürrer+, Phys. Rev. Research **2**, 023151)



chirp mass - mass ratio

precession spin - effective spin

Δ'_{net} vs posterior inconsistency



- Calculate Jensen–Shannon Distance between IMR and EOB samples
- Choose the maximum J-S Distance in samples of $q, M_{chirp}, \chi_{eff}, \chi_P$
- When the fraction of "good samples"<40%, the J-S Distance will be larger than most other events
- Waveform difference is not the only factor that can influence posterior consistency

Summary

- ❑ A waveform accuracy evaluation approach, free from NR simulations
  - Key idea: if two waveforms have significant difference, they can not be accurate at the same time
  - Drawback: can not determine which one is inaccurate, or both inaccurate

- ❑ BBH Real events
  - Only part of PE samples can pass our assessment; they are in the "ill-behaved" regions of parameter space (high spin and unequal mass)
  - Waveform difference has correlation with posterior sample consistency
  - Future 3rd-gen detectors: accuracy need to be improved 3+ orders of magnitude

More details: LIGO-G2200415, LIGO-P2200107

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