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Assessing the model waveform accuracy of gravitational waves

arXiv:2205.08448

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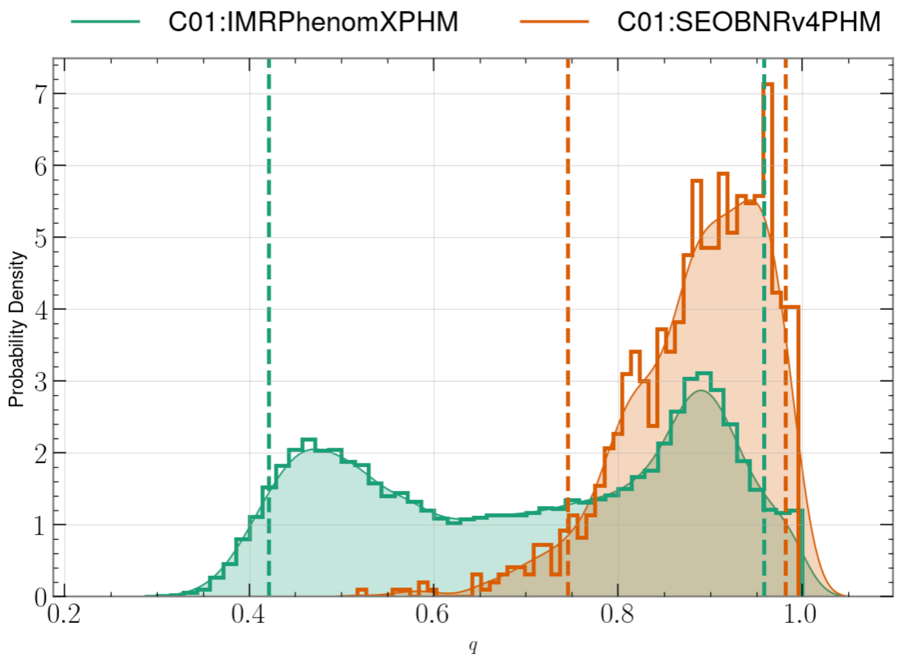
June 9 2022, Presentation for TianQin group

Motivations

- Waveform models always have errors
- Assessing waveform model accuracy needs numerical relativity (NR) simulations which is not quite accessible
- Waveform error induces systematic error in data analysis. The higher SNR is, the more accurate waveform models should be.

- In the latest GWTC-3 catalog, parameter estimation results showed difference in analysis driven by different waveform models: potential waveform systematics?



Mass ratio estimation for GW200129 given by analysis using different waveform models

Overview

❑ A new approach to evaluate GW waveform accuracy

- By looking into difference between two waveform models
- Free from the unknown true waveform (NR waveform), can be performed everywhere in the parameter space

❑ Applied to…

- GWTC-3 and GWTC-2.1 PE samples: How was IMRPhenomXPHM and SEOBNRv4PHM's performance?
- The relation between waveform difference and posterior difference
- Simulations: Good and bad regions in the parameter space & future accuracy requirements

Assessment of one waveform model

- Can detectors distinguish it from the real one?

- "Accurate enough": the detector can not distinguish it from the real waveform
- Construct such a waveform family for plus polarization: (Lindblom+, Phys. Rev. D **78**, 124020, 2008)

$$H_1^+(\lambda) = (1-\lambda)h_0^+ + \lambda h_1^+ = h_0^+ + \lambda \delta h_1^+, \quad 0 < \lambda < 1,$$ h0: real waveform, h1: model waveform

- Distinguishing waveforms <=> measuring λ

$$\sigma_\lambda^{-2} = \left(\frac{\partial h^+}{\partial \lambda} \mid \frac{\partial h^+}{\partial \lambda}\right) = (\delta h_1^+ \mid \delta h_1^+).$$

$$(a \mid b) = 4 \int_0^{+\infty} \frac{a^*(f) b(f)}{S_n(f)} df$$

- If the error of measuring λ is greater than its domain of definition (also the parametric distance between real and model waveforms), the detector can not distinguish

$$\|\delta h_1^+\|^2 = (\delta h_1^+ \mid \delta h_1^+) < 1.$$

- It shows: waveform error should lie within a unit ball in the inner-product space

Assessment of waveform pair

- Eliminate the unknown real waveform

$$\|\delta h_1^+\|^2 = (\delta h_1^+ \mid \delta h_1^+) < 1. \qquad \delta h_1^+ = h_1^+ - h_0^+$$

- The calculation of δh_1^+ needs the real waveform, which we don't know
- Use Numerical Relativity (NR) simulations as real waveform, but the number of NR simulations is limited
- Introduce another waveform model h_2, pair it with h_1

$$\begin{aligned} \Delta^+ &= \delta h_1^+ - \delta h_2^+ \\ &= (h_1^+ - h_0^+) - (h_2^+ - h_0^+) \\ &= h_1^+ - h_2^+. \end{aligned}$$

Real waveform is cancelled!

- Assume two waveforms are both accurate enough, we have

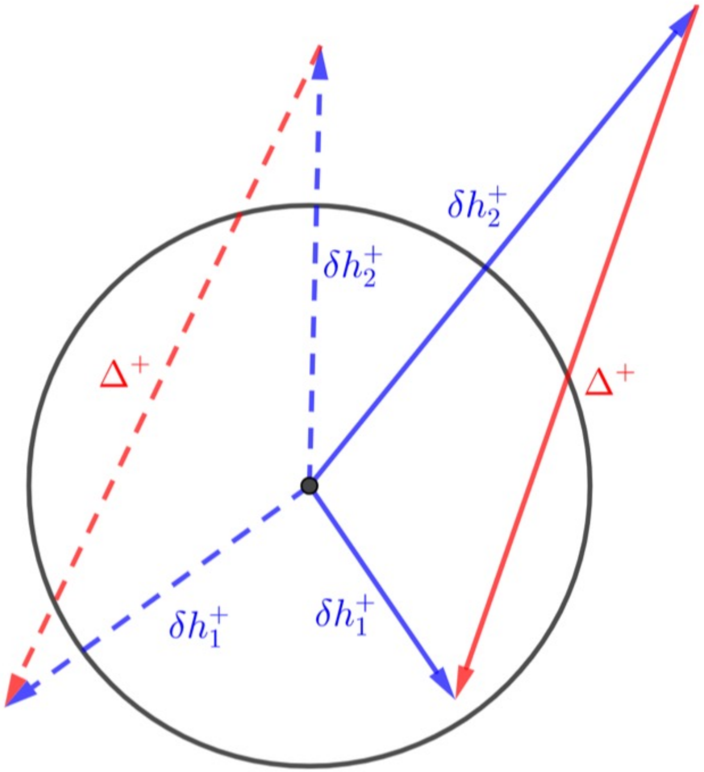
$$\|\Delta^+\| \leq \|\delta h_1^+\| + \|\delta h_2^+\| < 2.$$

- If we find $\|\Delta^+\| > 2$, at least one of the waveforms is not accurate enough. It's a necessary condition of "a pair of waveform models are both accurate".

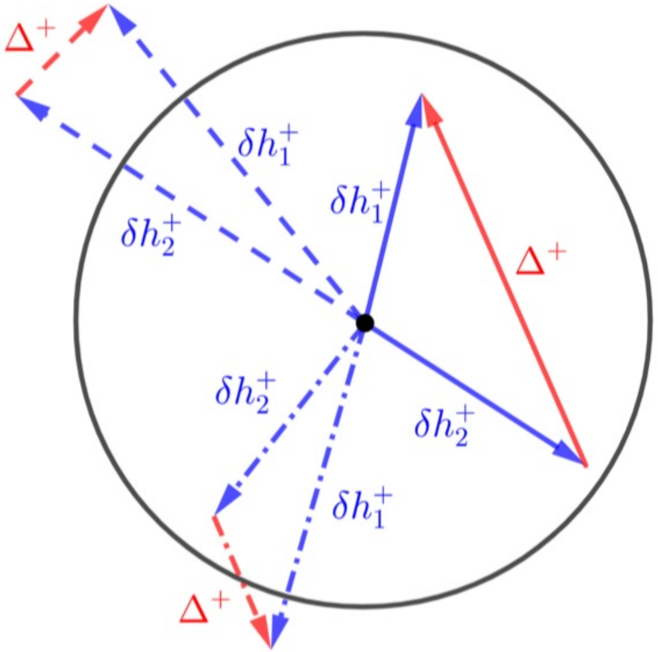
Assessment of waveform pair

- An illustration of all possible cases

- If we find $|| \Delta^+ || > 2$, at least one of the waveforms is not accurate enough
- Which one?



Case I: $\Delta^+ > 2$



Case II: $\Delta^+ < 2$

Assessment of waveform pair

- Normalization & Relations with overlap

- $\Delta^+ = (h_1^+ - h_2^+ | h_1^+ - h_2^+)$, is proportional to the amplitude of GWs. Louder events tend to have larger Δ^+. We want to eliminate the impact of SNR and investigate waveform model's intrinsic performance in some specific parameter regions.
- Normalize Δ^+ with SNR (geometric mean of SNRs of two waveforms, i.e. $\sqrt{\rho_1\rho_2}$.)

$$\|\Delta^+_{\mathrm{SNR}=1}\|^2 = \frac{(h_1^+ - h_2^+ | h_1^+ - h_2^+)}{\sqrt{(h_1^+|h_1^+)(h_2^+|h_2^+)}} \qquad \|\Delta^+_{\mathrm{SNR}=\rho_0}\| = \rho_0 \|\Delta^+_{\mathrm{SNR}=1}\|$$

- Compared to overlap which is widely-used in the waveform community

$$\mathcal{O}(h_1, h_2) = \Re \frac{(h_1|h_2)}{\sqrt{(h_1|h_1)(h_2|h_2)}}, \qquad \|\Delta^+_{\mathrm{SNR}=1}\|^2 = \frac{\rho_1^+}{\rho_2^+} + \frac{\rho_2^+}{\rho_1^+} - 2\mathcal{O}(h_1^+, h_2^+)$$

- Δ^+ analysis is consistent with overlap method. But Δ^+ has a clear upper limit 2.

Assessment of waveform pair

$$\|\Delta^{+}\| \leq \|\delta h_1^{+}\| + \|\delta h_2^{+}\| < 2.$$

- Extend to detector response:

$$\|\Delta\| \leq \|\delta h_1\| + \|\delta h_2\| < 2(|F_+| + |F_\times|).$$

- Extend to detector network:

$$\mathbf{C} = (\mathbf{D}|\mathbf{B}) \Rightarrow C_{jk} = \sum_{p=1}^{n} (D_{jp} \mid B_{pk})$$

$$\|\boldsymbol{\Delta}_{\rm det}\| = (\delta\mathbf{h}^{\mathbf{T}}|\delta\mathbf{h}) = \sum_k (\delta h^{(k)}|\delta h^{(k)})$$
$$= \sum_k \left(\Delta^{(k)}\right)^2 < 2\sum_k (|F_+^{(k)}| + |F_\times^{(k)}|).$$

- To sum up:

$$\Delta'^{(k)} = \frac{\Delta^{(k)}}{|F_+^{(k)}| + |F_\times^{(k)}|}, \quad \boldsymbol{\Delta}'_{\rm det} = \frac{\boldsymbol{\Delta}_{\rm det}}{\sum_k (|F_+^{(k)}| + |F_\times^{(k)}|)}$$

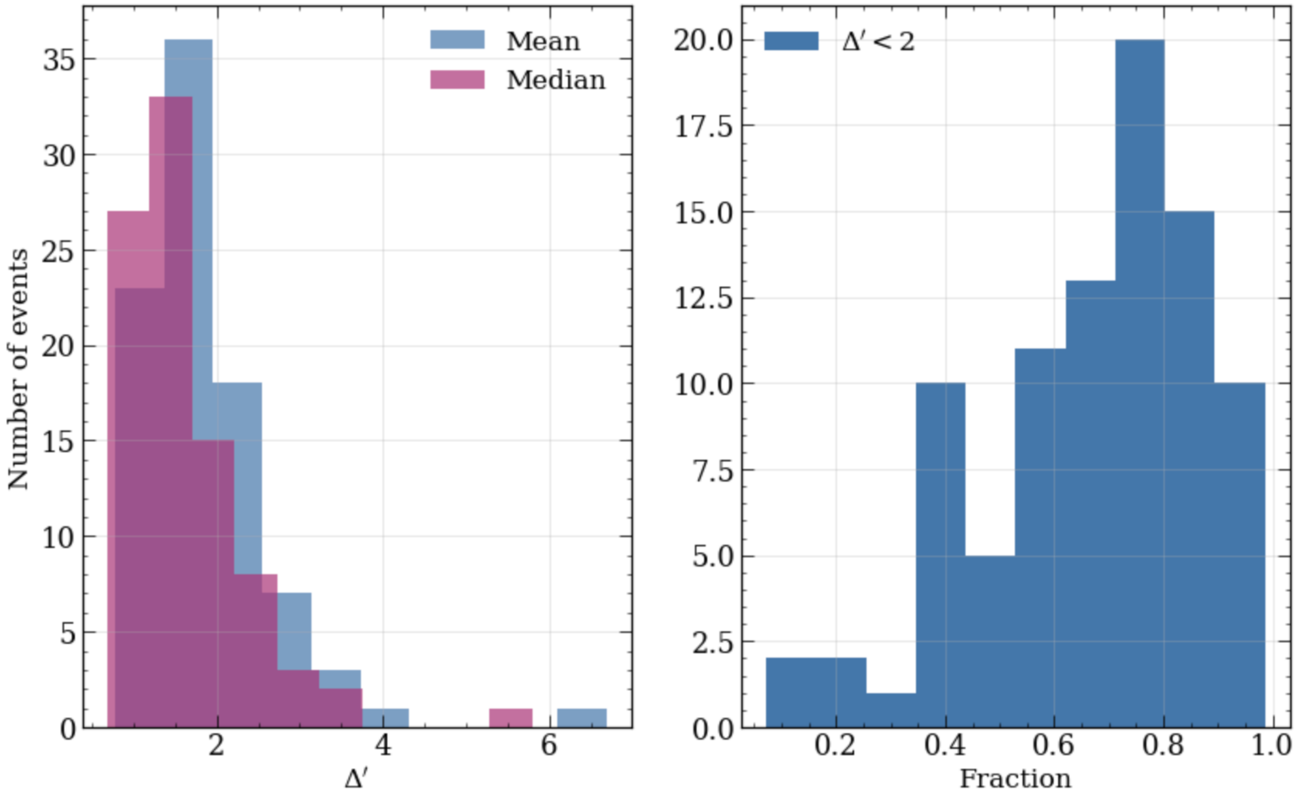
They should be less than 2 if both models are accurate!

Applying to PE samples

- Overview: histograms

- For each event, calculate $\Delta_{net}^{'}$ for the mixed posterior samples from IMRPhenomXPHM & SEOBNRv4PHM
- Calculate mean, median of $\Delta_{net}^{'}$ for each event (left panel)
- Calculate fraction of $\Delta_{net}^{'} < 2$ samples for each event (right panel)

- There are several events having "worse" performance compared to the others

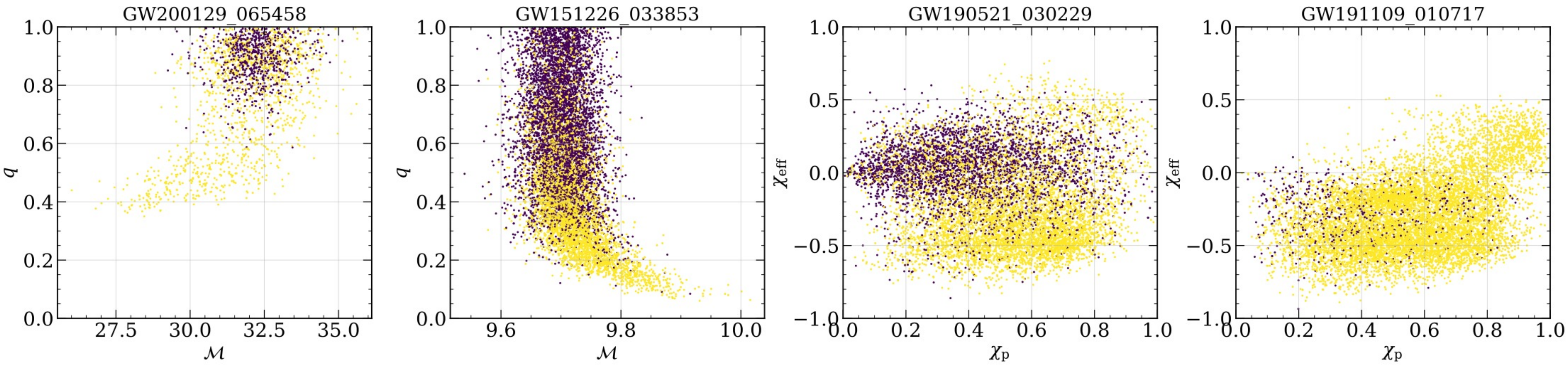


$$\Delta'_{\rm det} = \frac{\Delta_{\rm det}}{\sum_k (|F_+^{(k)}| + |F_\times^{(k)}|)}$$

Applying to PE samples

- Overview: distribution in mass and spins

- Yellow points: $\Delta'_{net} > 2$ samples
- Purple points: $\Delta'_{net} < 2$ samples
- Accuracy becomes worse when mass ratio decreases or spins increase



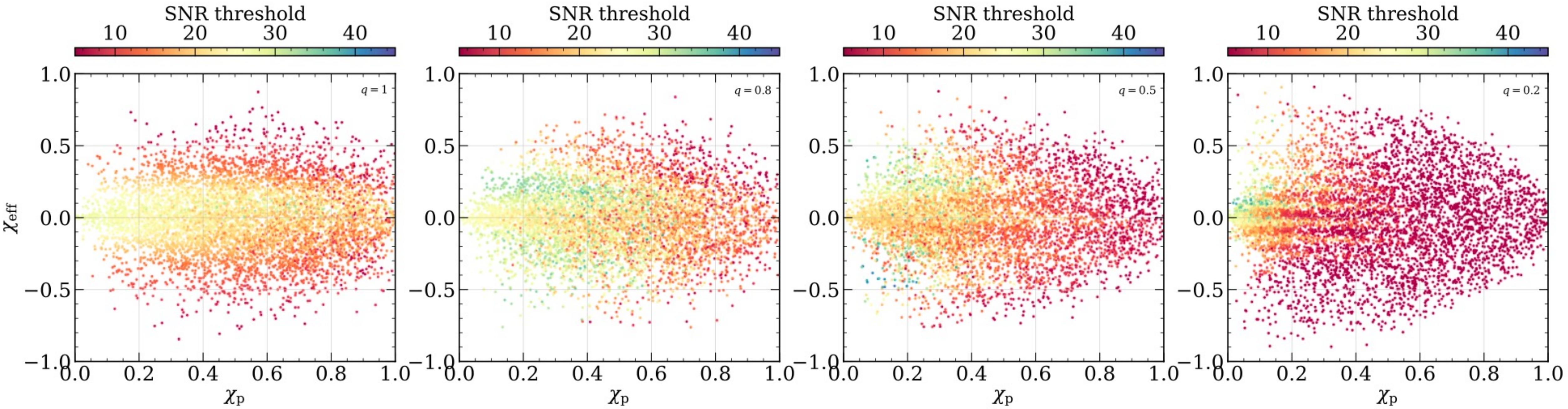
chirp mass - mass ratio

precession spin - effective spin

BBH Simulations

- IMRPhenomXPHM and SEOBNRv4PHM

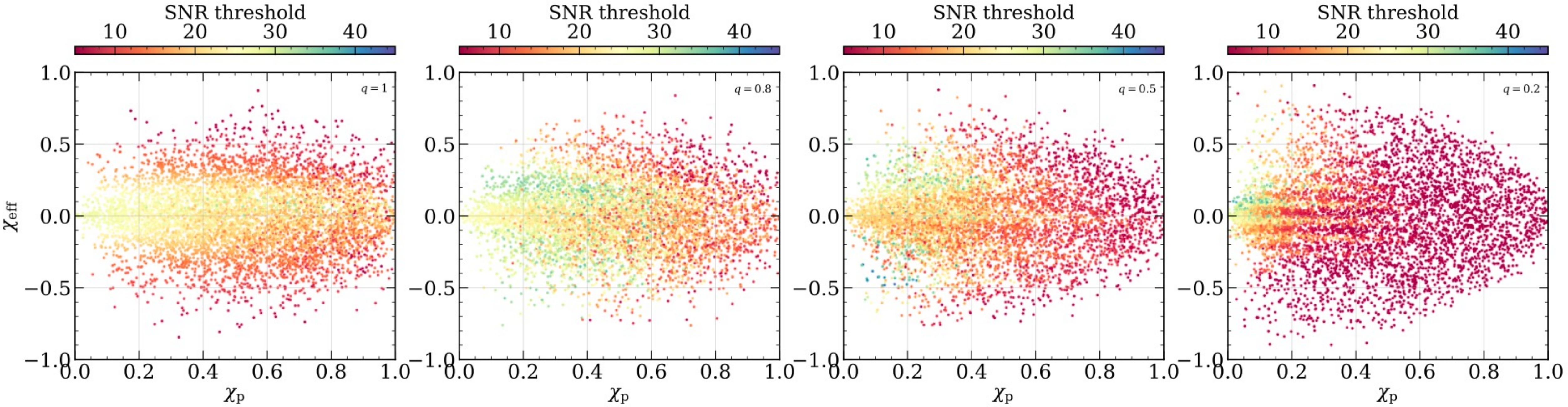
- $m_1 = 30M_{\odot}, q = 1, 0.8, 0.5, 0.2$
- Spins are randomly generated (isotropic, uniform between 0 and 1)
- SNR threshold: SNR when waveform difference reaches upper limit 2



BBH Simulations

- IMRPhenomXPHM and SEOBNRv4PHM

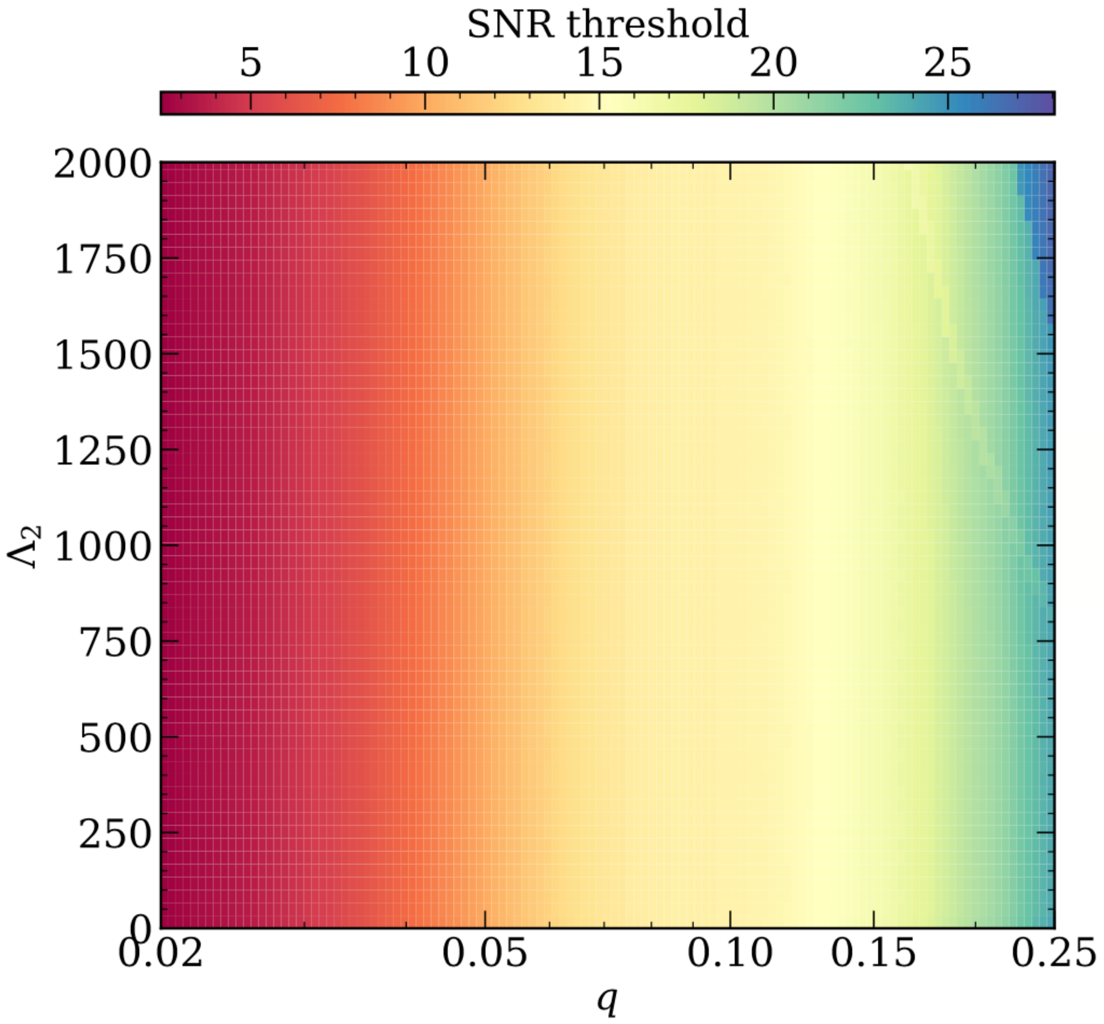
- Waveform accuracy deteriorates as spin goes up or mass ratio goes down
- In some cases, SNR threshold drops below 5
- Using $\Delta \propto SNR$, for 3rd-gen detectors (SNR 30~1000), the model mismatch from true waveform should be improved by 3-4+ orders of magnitude (consistent with Pürrer+, Phys. Rev. Research **2**, 023151)



NSBH simulations

IMRPhenomNSBH and SEOBNRv4_ROM_NRTidalv2_NSBH

- $m_2 = 1.4M_\odot, q \in [0.02, 0.25], \Lambda_2 \in [0,2000]$
- We assume zero-spin, as both models are calibrated with non-spin simulations



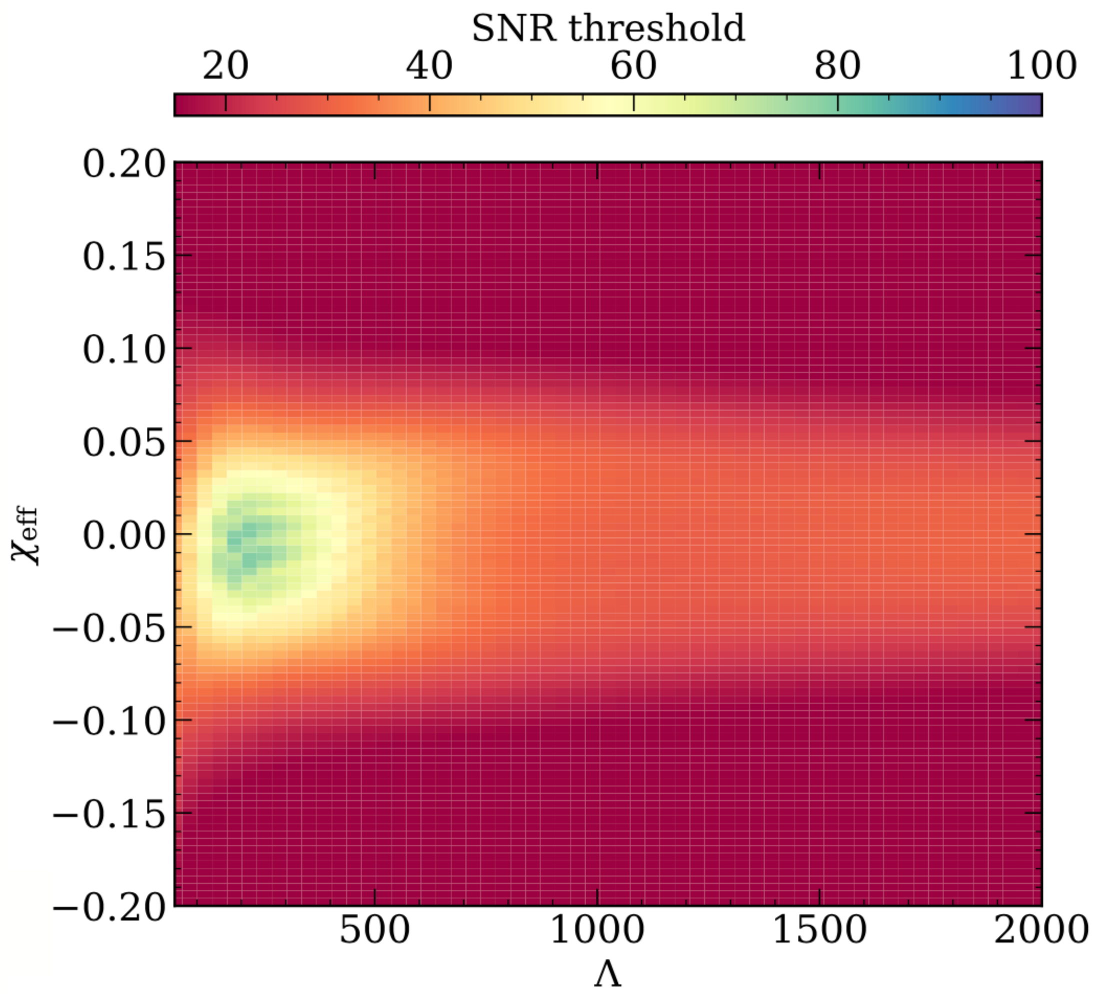
- Mass ratio has more impacts on waveform accuracy because both waveforms use NRTidal to model matter effects
- Waveform accuracy should also be improved for future high SNR observations , or when more complex physical effects are included (spins, higher modes or eccentricity etc)

BNS simulations

IMRPhenomPv2_NRTidalv2 and SEOBNRv4T_surrogate

- $m_1 = m_2 = 1.4M_{\odot}, S_1 = S_2, \Lambda_1 = \Lambda_2$
- Aligned spin $|S_1| < 0.2,\ \Lambda_1 \in [0,2000]$

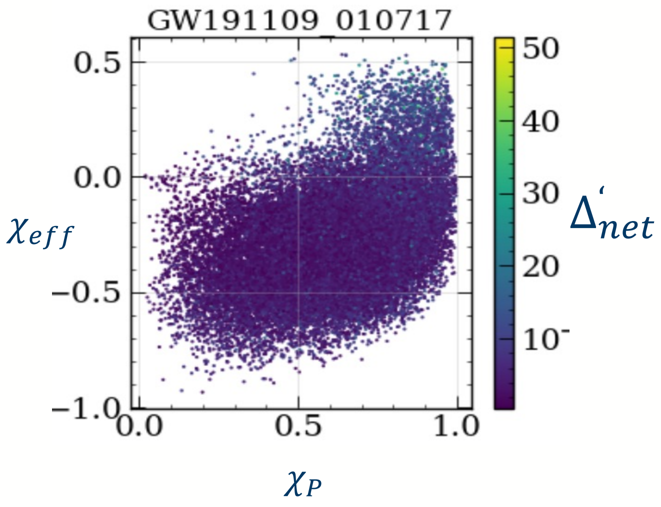
- Two waveform models agree with each other quite well in $\Lambda < 500, |S| < 0.05$, this is the region that coincides with our current knowledge of neutron star
- Waveform accuracy should be improved for future high SNR observations, or when more complex physical effects are included (high spin scenario, precession effects etc)

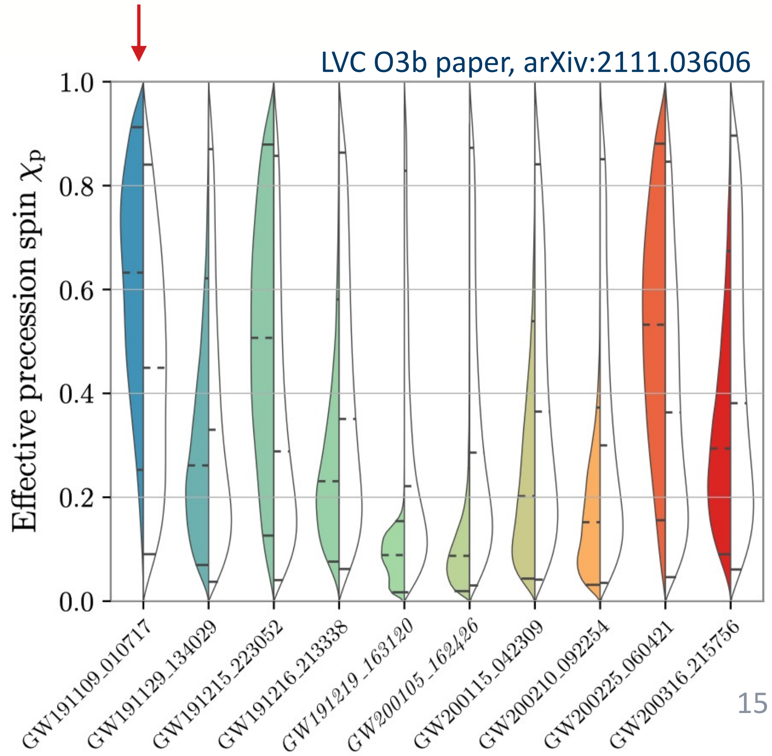


Applying to O3b PE samples

GW191109: the largest waveform difference in our analysis

- High mass BBH (Mchirp ~ 50Msun)
- Many bizarre behaviors in testing GR (arXiv:2112.06861)
- Has the smallest (negative) χ_{eff} in O3b catalog, large χ_P, "where waveform differences may be expected" – GWTC3 paper, also arXiv:2010.05830, arXiv:2106.06492
- Also has large difference in waveforms means the two waveforms can not be both accurate

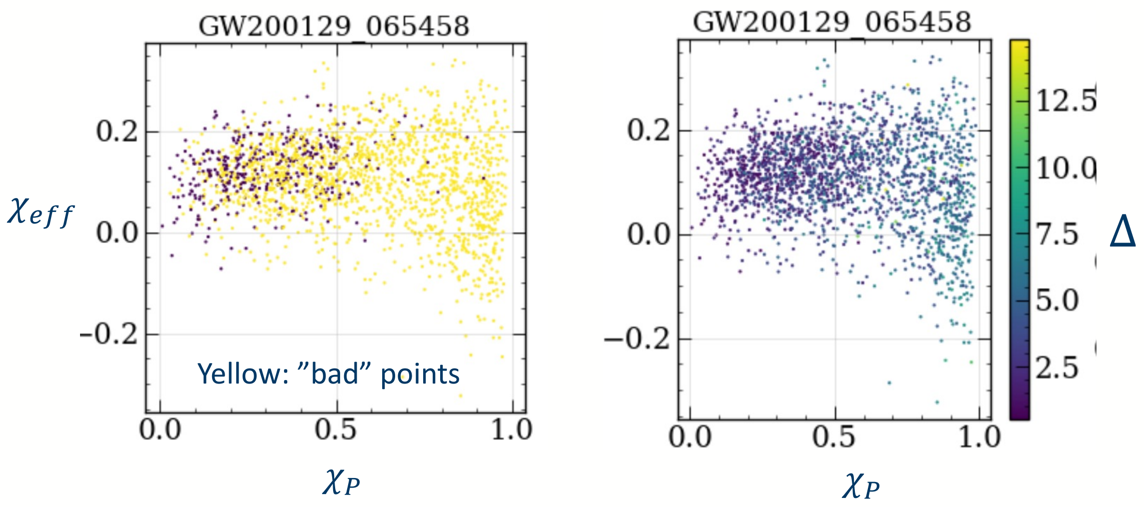


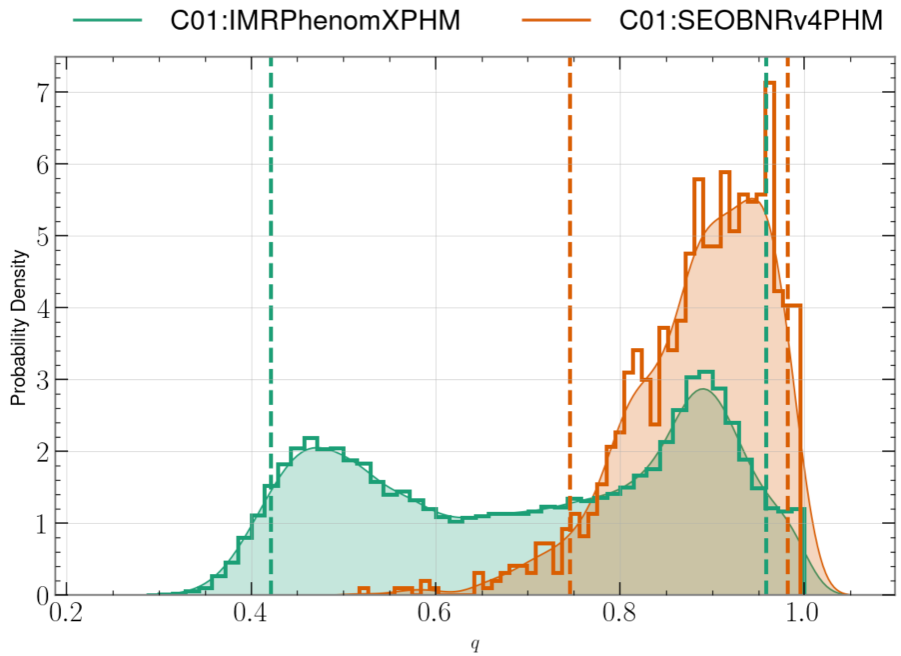


Applying to O3b PE samples

GW200129

- The highest SNR in O3b Catalog (SNR~26.8)
- The highest inferred χ_P
- PE results showed difference between two waveform models



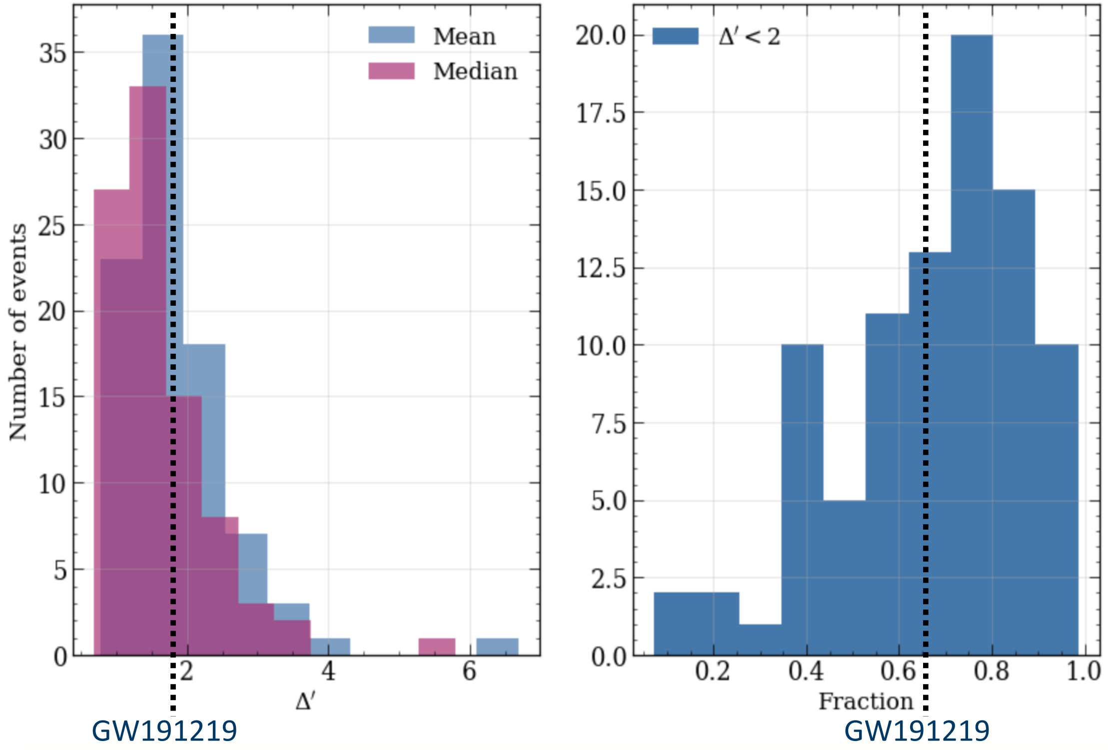


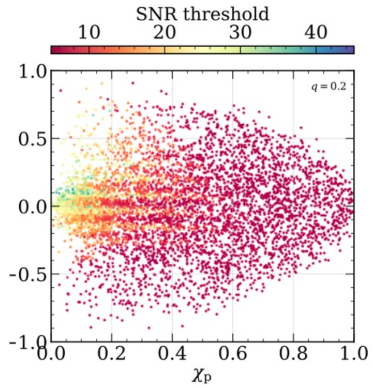
Applying to O3b PE samples

- “Extreme”-mass-ratio event GW191219

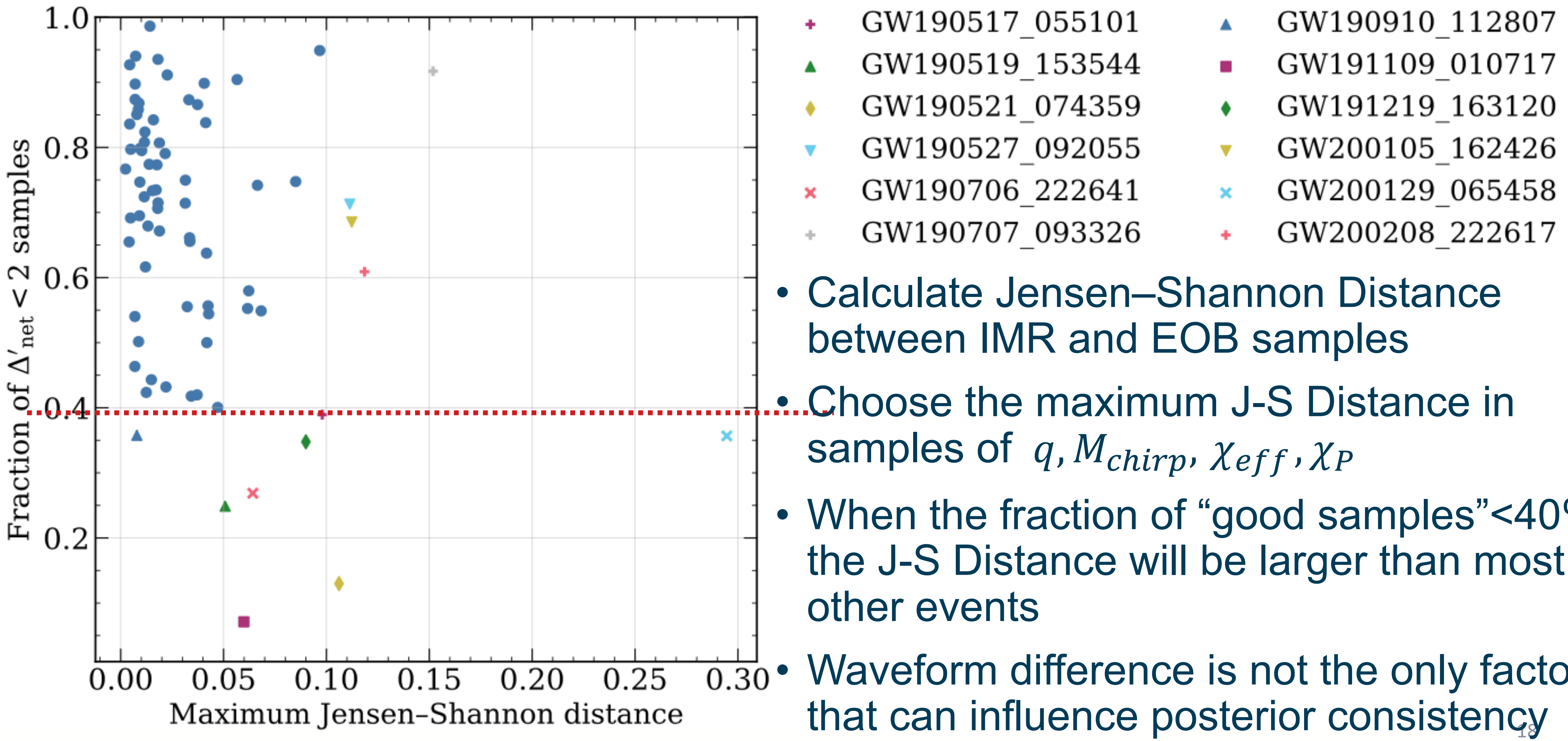
GW191219

- The lowest mass ratio to-date, out of the waveform calibration range
- The smallest χ_P in O3b, $\chi_{eff} = 0.00^{+0.07}_{-0.09}$
- Mean value of Δ'_{net}: 1.77 (< 2)
- Fraction of Δ'_{net} <2 samples: 0.62
- Waveform performance is “not too bad” compared to other events
- Spin is more problematic than mass ratio





Δ'_{net} vs posterior inconsistency



- Calculate Jensen–Shannon Distance between IMR and EOB samples
- Choose the maximum J-S Distance in samples of $q, M_{chirp}, \chi_{eff}, \chi_P$
- When the fraction of “good samples”<40%, the J-S Distance will be larger than most other events
- Waveform difference is not the only factor that can influence posterior consistency

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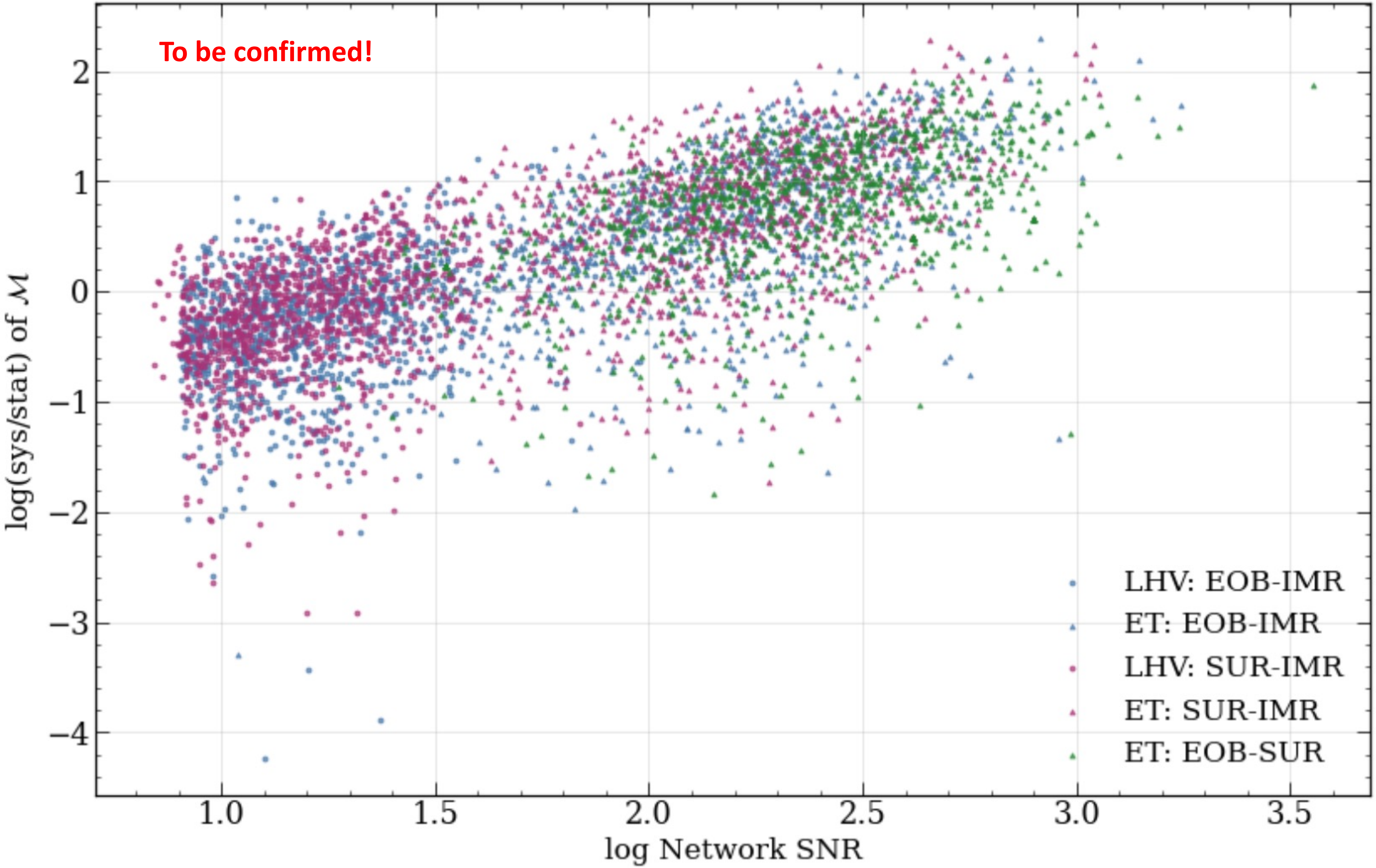
Summary arXiv:2205.08448

- A waveform accuracy evaluation approach, free from NR simulations
  - Key idea: if two waveforms have significant difference, they can not be accurate at the same time
  - Drawback: can not determine which one is inaccurate, or both inaccurate
- BBH Real events & simulations
  - Only part of PE samples can pass our assessment; they are in the "well-behaved" regions of parameter space (low spin and equal mass)
  - Waveform difference has correlation with posterior sample consistency
  - Future 3rd-gen detectors: accuracy need to be improved 3+ orders of magnitude
- NSBH/BNS simulations
  - Mass ratio is the main factor that influences NSBH waveform accuracy, as complex spin effects are not included
  - Two BNS waveform agree with each other quite well in the region that coincides with our current knowledge of neutron star
  - Both types of waveform need improvements
- Implications and other applications
  - Need to be careful about waveform systematics. Waveform difference check can be part of PE workflow in future LVK data analysis
  - Check other waveform models in the future, like eccentric binaries
  - Account for unknown true waveform in detectability forecast


To be confirmed!
log(sys/stat) of $\mathcal{M}$
log Network SNR
LHV: EOB-IMR
ET: EOB-IMR
LHV: SUR-IMR
ET: SUR-IMR
ET: EOB-SUR